Word Problems 24: Mixed-Rate Problems #21

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Abstract

In this algebra word problem note, we use the Scheme to solve our twenty-first attempt at what I refer to as a 'mixed-rate problem'.

1 Introduction

Our first problem is a fairly knotty problem involving mixtures and their profits. In mixture problems, cost is usually an invariant of mixing, but profit is arbitrary and can be set arbitrarily at any time. In the the first problem, we'll make some assumptions about the affixed profits on the component parts.

2 Word Problem #24.1

[1] Two kinds of Vodka are mixed in the ratio 1:2 and 2:1 and they are sold fetching the profit 10% and 20% respectively. If the vodkas are mixed in equal ratio and the individual profit percent on them are increased by 4/3 and 5/3 times respectively, then the mixture will fetch the profit of

A. 18%
B. 20%
C. 21%
D. 23%
E. Cannot be determined

3 Solution 24.1.1: Conceptualizing the Problem

The first thing to note in this problem is the irelevant information. That is the specific ratios by which the component vodkas are mixed to form Mix 1 at 10% profit and Mix 2 at 20% profit. The reason we do not need to now these ratios is simply because the question asks us nothing about the costs or profits of the

[1] Found at http://gmatclub.com/forum/two-kinds-of-vodka-are-mixed-in-the-ratio-1-2-and-2-1-and-113897.html.

component vodkas. What we are asked to find is the percentage profit on the third mix, Mix 3, made by adding Mix 1 and Mix 2 together.

Before we consider the 'profit factors', where do we stand at the moment? Let's make a figure.

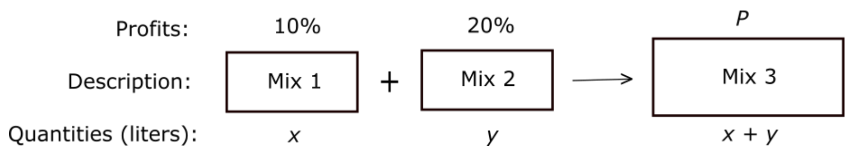

Figure 1. We have chosen the quantities to be in liters, but any volume unit would do as well.

From our long experience with these kinds of percentage problems, we know that the profit percentage of Mix 3 must be something between 10% and 20%. In fact, it will be in the exact middle between them (their average of 15%) if we combine the mixes in equal amounts. But now we're asked to consider the effect of applying arbitrary multiplication factors to these mixtures on the profit percentage of Mix 3. Next figure, please!

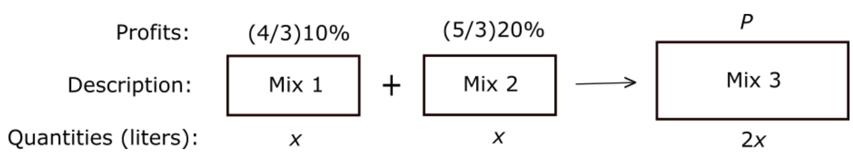

Figure 2. The whole problem boils down to dealing with this simple figure.

We need only balance on the conserved values of profits (not their percentages).

$$(4/3)10x + (5/3)20x = P \cdot 2x\,. \qquad (1)$$

This has solution $P = 23.3\%$. So, the answer is D.

4 Word Problem #24.2

[2] A man had a 10-gallon keg of wine and a jug. One day, he drew off a jugful of wine and filled up the keg with water. Later on, when the wine and water had got thoroughly mixed, he drew off another jugful and again filled up the keg with water. The keg then contained equal quantities of wine and water. What was the capacity of the jug?

[2]Found at http://www.theproblemsite.com/problems/high-school-math/2008/mixture-problem.

5 Solution 24.2.1: Conceptualizing the Problem

We begin with the reasonable asumption that the jug has less volume than the keg. Now, to the figure.

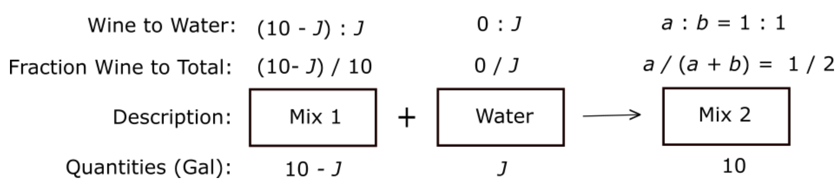

Figure 3. Because both wine and water are treated as conserved, we can balance on either one of them.

Let's balance on wine in this 'before and after' process:

$$\frac{10-J}{10}(10-J) + \frac{0}{J}J = \frac{1}{2}\,10\,. \qquad (2a)$$

From this we get the quadratic in J:

$$(10-J)^2 = 50\,, \qquad (2b)$$

which has the solution consistent with our assumptions,

$$J = 5(2-\sqrt{2}) \approx 2.93 \text{ [gallons]}. \qquad (3)$$

6 Word Problem #24.3

[3] Your recipe calls for 2 cups of regular flour and one-half tablespoon of baking powder. But your pantry only has a cup of regular flour and a cup of self-rising flour, which is 4% baking powder. How much regular flour should we add to the self-rising flour to get a mixture with the desired concentration of baking powder?

7 Solution 24.3.1: Conceptualizing the Problem

There are a number of issues to deal with prior to writng conservation equations. First, let's get ourselves a single measuring unit. There are 16 tablespoons in one cup. Therefore, one-half tablespoon is 1/32 cup. Second, The question tells us to maintain a certain 'concentration', but I prefer to think of it as a proportion. That is, the final mix is to have the same ratio of regular flour to baking powder as was called for in the recipe. This is just common sense. Let

[3] http://iws.collin.edu/dkatz/Intermediate_Algebra/Mixture_Problems.pdf.

R be the percentage of regular flour in the final mix. Let B be the percentage of baking powder in the final mix.[4] Then

$$\frac{R}{B} = \frac{32\,\text{tablespoons}}{\frac{1}{2}\,\text{tablespoon}} = \frac{64}{1}\,. \tag{4}$$

My third issue is to make sense of the claim that self-rising flour is 4% baking powder. What is the other 96% of the flour? For simplicity sake, I'm going to assume that it's all regular flour.

Now, if I understand the original problem correctly, we use all the self-rising flour and some of the regular flour, but none of the baking poweder. That is a simple problem, which the reader can solve if he or she wants to.

However, I propose solving a more interesting problem: Knowing me, I'd mix the one cup of regular four, the 1/2 tablespoon of baking powder, and the 1 cup of self-rising flour (mistaking it for regular flour) and then realize my mistake of thinking that the self-rising flour was regular. Then I would realize that the self-rising flour already has baking powder in it and then I have to calculate how much more regular flour I must borrow from my neighbor to add into the mixing bowl to maintain proper proportion given in (4).

Question: How much more regular flour should I add to the mixing bowl?

As you can see in the figure below, I have placed an x below the regular-flour rectangle. We'll solve for x and then subtract off 1 cup from it, since we already have 1 cup of regular flour in the mix.

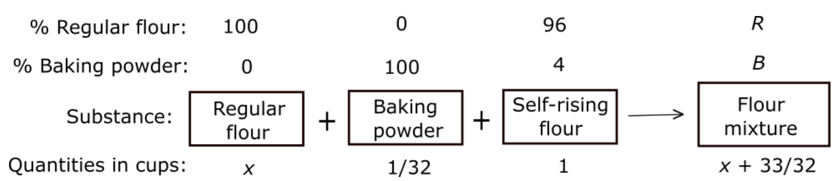

Figure 4. I have already included the conservation of dry ingredients (by volume) in the final flour mix.

Let's balance on regular flour first.

$$1.00(x) + 0(1/32) + .96(1) = \frac{R}{100}(x + 33/32)\,. \tag{5}$$

Now, let's balance on baking powder.

$$0(x) + 1.00(1/32) + .04(1) = \frac{B}{100}(x + 33/32)\,. \tag{6}$$

[4]Taking the ratio of the percent R to the percent P gives the same value as taking the actual amount of regular wheat in the final mix to the actual amount of baking powder in the final mix.

Since we have to connect back to (4), let's divide (5) by (6), and simplify as we go, to get

$$\frac{x + .96}{1/32 + .04} = \frac{R}{B} = \frac{64}{1} \,. \tag{7}$$

The solution is $x = 3.6$ cups. That means that we must add 2.6 more cups of regular flour into the mix to restore the proper ratio of regular flour to baking powder as given in the baking instructions.

8 Word Problem #24.4

[5] Solution Y is 30 percent liquid X and 70 percent water. If 2 kilograms of water evaporate from 8 kilograms of solutions Y and 2 kilograms of solution Y are added to the remaining 6 kilograms of liquid, what percent of this new liquid solution is liquid X?

9 Solution 24.4.1: Conceptualizing the Problem

Without loss of generality, I will assume that we have an 8 Kg sample of liquid Y in an large beaker. I will solve for the answer in two steps. First, I will find the ratio of X to water in the beaker after the evaporation as Step 1. Then for Step 2, I will find the ratio of X to water in the beaker after we add into it 2 more Kg of solution Y.

STEP 1: Let P_1 and W_1 be the percentages of X and water, respectively, in the beaker fluid (Y_1) after evaporation.

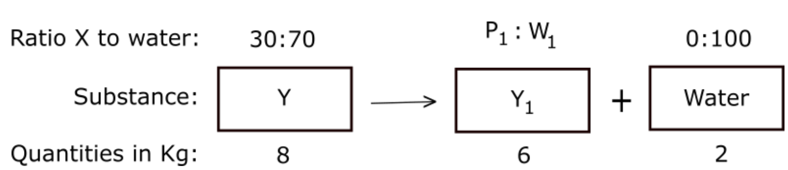

Figure 5. The setup for Step 1.

Time to balance on quantities of X across the process.

$$.30(8) = \frac{P_1}{100}(6) \,, \tag{8}$$

which has solution $P_1 = 40\%$.

STEP 2: Let P_2 and W_2 be the percentages of X and water, respectively, in the beaker fluid (Y_2) after the addition of 2 Kg more fluid Y.

[5] Found at http://www.beatthegmat.com/hard-solutions-mixture-problem-t69704.html.

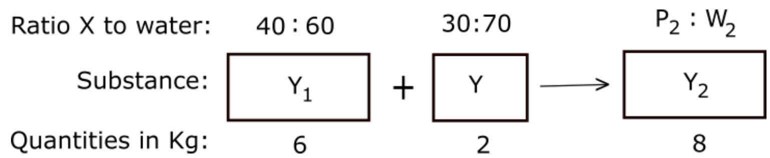

Figure 6. The setup for step 2.

Once again, balance on quantities of X across the process.

$$.40(6) + .30(2) = \frac{P_2}{100}(8)\,, \tag{9}$$

which has solution $P_2 = 37.5\%$.

10 Conclusion

I've reached the end of the interesting mixed-rate problem I wanted to solve. There are many of types of word problems for me to start on soon, I hope.