Word Problems 26

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Abstract

In this algebra word problem note, we use the Scheme to solve a variety of problem that may or may not include 'mixed-rate' types.

1 Introduction

We venture to all kinds of words problem for my foreseeable future, beginning with double-ratio problem.

2 Word Problem #26.1

A, B, and C invested money in ratios 6 : 4 : 9 and their interests from the first cycle were in the ratios 3 : 2 : 5. Assuming that their interests were proportional to the times of their investments, show that their investment times were in the ratios 9 : 9 : 10.

3 Solution 26.1.1: Conceptualizing the Problem

First, the investment principal ratios is given as

$$P_1 : P_2 : P_3 :: 6 : 4 : 9\,. \tag{1}$$

The interest on an investment is given as its principal times the time it is invested, or $I = PT$.Therefore, we have that

$$P_1T_1 : P_2T_2 : P_3T_3 :: 3 : 2 : 5\,. \tag{2}$$

Therefore, we can write from the first couple ratios of (1) and same for (2):

$$\frac{P_1}{P_2} = \frac{6}{4} \quad \text{and} \quad \frac{P_1T_1}{P_2T_2} = \frac{3}{2}\,, \tag{3}$$

from which we get

$$\frac{T_1}{T_2} = \frac{1}{1}\,, \tag{4}$$

Put into ratio form, we get

$$T_1 : T_2 :: 1 : 1\,. \tag{5}$$

Now we repeat this procedure for indices 1 and 3, say, to get

$$\frac{P_1}{P_3} = \frac{6}{9} \quad \text{and} \quad \frac{P_1 T_1}{P_3 T_3} = \frac{3}{5}\,, \tag{6}$$

from which we get

$$\frac{T_1}{T_3} = \frac{27}{30} = \frac{9}{10}\,, \tag{7}$$

Put into ratio form, we get

$$T_1 : T_3 :: 9 : 10\,. \tag{8}$$

Putting (5) and (8) together, we get

$$T_1 : T_2 : T_3 :: 9 : 9 : 10\,. \tag{9}$$

4 Word Problem #26.2

The ratios, by weight, of four ingredients A, B, C, and D of a certain mixture is $4 : 7 : 8 : 12$. The mixture will be changed so that the ratio of A to C is quadrupled and the ratio of A to D is decreased. The ratio of A to B is held constant. If B constitutes 20% of the weight of the new mixture, by approximately by what percent will the ratio of A to D be decreased?

A. 15%
B. 25%
C. 35%
D. 45%
E. 55%

5 Solution 26.2.1: Conceptualizing the Problem

Mastering the previous problem is probably useful for understanding how to do this one. Just the same, this problem has quite a few moving parts of its own.

We start with an initial state of

$$A : B : C : D \quad \longleftrightarrow \quad 4 : 7 : 8 : 12\,. \tag{10}$$

On our way to the goal, we need to find the final serial ratios

$$A' : B' : C' : D' \quad \longleftrightarrow \quad ? :? :? :?\,, \tag{11}$$

using the given information. Let's go one change at a time. 1) A to C is quadrupled:

$$A' : B' : C' : D' \quad \longleftrightarrow \quad 16 :? : 8 :?\,, \tag{12}$$

2) The ratio of A to D is decreased. We'll hold off on this jem. 3) The ratio of A to B is held constant. Now, we're getting somewhere. $A' : B' :: A : B$. So, $16 : B' :: 4 : 7$. Solving this simple proportion gives us $B' = 28$.

$$A' : B' : C' : D' \quad \longleftrightarrow \quad 16 : 28 : 8 : x\,, \tag{13}$$

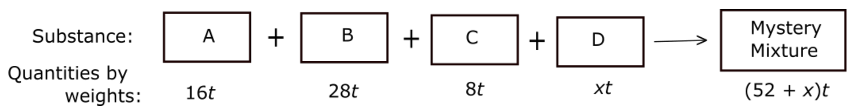

Figure 1. The new mixture, nearly specified. We don't know t and we don't need to solve for it either, but including it means we keep the ratios right. What we really need to know is x.

Once we know x, we can calculate how the ratio of A/D has decreased. This x is constrained by the fact that B constitutes 20% of the weight of the new mixture, or, put another way: B is .20 of the total weight of the new mixture.

Figure 1 show us the ingredients to make the new mixture. The parameter t is unknwn, except that it must be a positive real number. It will cancel in the relation given below:

$$28t = 0.20(52 + x)t\,. \tag{14}$$

After cancelling the t's, we get $x = 88$. This bring our serial ratio up to date:

$$A' : B' : C' : D' \quad \longleftrightarrow \quad 16 : 28 : 8 : 88\,, \tag{15}$$

Now for the precent change of the ratio of $A : D$.

$$\text{Percent Change} = \frac{\text{new ratio} - \text{old ratio}}{\text{old ratio}} \times 100\%\,. \tag{16}$$

Substituting in the values

$$\begin{aligned} \text{Percent Change} &= \frac{A'/D' - A/D}{A/D} \times 100\% \\ &= \frac{16/88 - 4/12}{4/12} \times 100\% \\ &= -.4545\,. \end{aligned} \tag{17}$$

This gives us a decrease in the ratio A/D of about 45% (Ans. D).

6 Word Problem #26.3

In a particular season, a team won 50% of its first 60 games and 80% of the remaining games. If the team won 60% of all of its games that season, how many games did the team play?

7 Solution 26.3.1: Conceptualizing the Problem

This is a fairly simple mixed-rate problem. Let G be the total number of games played.

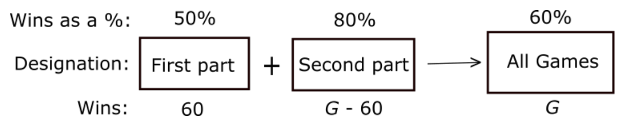

Figure 2. Standard mixed-rate problem. We'll convert percentages to decimals to balance on wins on both sides.

Time to balance on wins on both sides

$$.50(60) + .80(G - 60) = .60G\,, \tag{18}$$

which has the solution $G = 90$.

8 Word Problem #26.4

Question 250998:[1] While hiking up and then down a trail, Rolf spent 60% of his time hiking uphill and 40% hiking back down. If he averaged 2 mph uphill, what was his average speed round trip?

9 Solution 26.4.1: Conceptualizing the Problem

Let $\overline{v}$ stand for average speed and T stand for the total time of the hike.

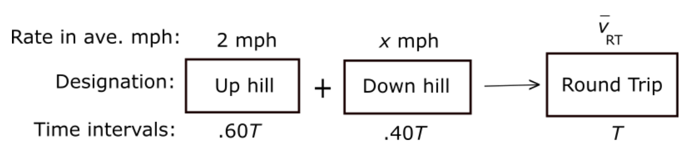

Figure 3. Rolf's hike parsed as totals and parts.

In the above figure, I used x for the average speed going down, but now I'll use $\overline{v}_{\downarrow}$. If we balance on the total distance across the process, we get

$$\overline{v}_{\uparrow}(.60T) + \overline{v}_{\downarrow}(.40T) = \overline{v}_{\mathrm{RT}}T \tag{19}$$

where 'RT' is short for 'round trip'. Solving for $\overline{v}_{\mathrm{RT}}$, we get

$$\overline{v}_{\mathrm{RT}} = \overline{v}_{\uparrow}(.60) + \overline{v}_{\downarrow}(.40)\,. \tag{20}$$

[1] Found at https://www.algebra.com/algebra.

We were given two other pieces of information that we need right now. First, that $\overline{v}_{\uparrow} = 2\,\text{mph}$, and second, that the uphill distance is the same as the downhill distance: $D_{\uparrow} = D_{\downarrow}$, giving us the constitutive relation

$$\overline{v}_{\uparrow}(.60T) = \overline{v}_{\downarrow}(.40T)\,, \tag{21}$$

from which we get that

$$\overline{v}_{\downarrow} = \frac{3}{2}\,\overline{v}_{\uparrow} = 3[\text{mph}]\,. \tag{22}$$

Substituting this information into (20) gives

$$\overline{v}_{\text{RT}} = 2[\text{mph}](.60) + 3[\text{mph}](.40) = 2.4[\text{mph}]\,. \tag{23}$$

10 Conclusion

If you can think of the last problem as composed of two different parts characterized by different rates (speeds), then you can think of this kind of kinematics problem as a 'mixture' problem, though I prefer to refer to it as a 'mixed-rate' problem.