Word Problems 27

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Abstract

In this algebra word problem note, we use the Scheme to solve a variety of problem that may or may not include 'mixed-rate' types.

1 Introduction

We venture to all kinds of words problem for my foreseeable future, beginning with double-ratio problem.

2 Word Problem #27.1

[1] A cylindrical glass is filled with three different juice mixtures. First, 1/4 of the glass is filled with a mixture that is half apple juice and half orange juice. Then, the glass is filled to the 80% mark with a mixture that has twice as much orange juice as apple juice and twice as much apple juice as pineapple juice. The remainder of the glass is filled with pineapple juice. What percent of the final mixture is apple juice? [Ans. 28.8%]

3 Solution 27.1.1: Conceptualizing the Problem

This is a typical double-proportion problem, in which we have 'apple juice : orange juice : pineapple juice', or A:O:P. We just need to be careful when interpreting the information given to us.

First, we'll invent on the fly a new volume called the 'glass full'. The volumic quantities given below the rectangle are given as fractional amount of 1 glass full. Mix 2 is added to the glass up to the 80% amount. This means that the quantity of Mix 2 is given as $.80 - 1/4 = 11/20$, since it already had 1/4 glass full of Mix 1 in it. I converted $1 : 2 : 1/2$ to $2 : 4 : 1$ since it is easier for me to deal with, yet leaves the ratios the same.

Let's look at the graphic now.

[1]Found at http://advancedmathtutoring.com/a-hard-sat-math-mixture-word-problem.

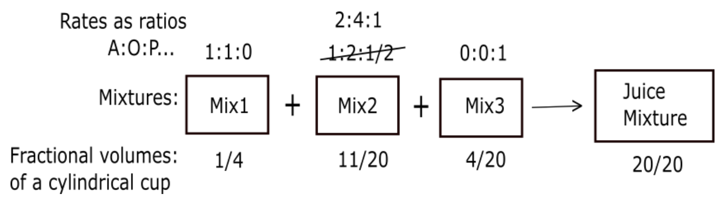

Figure 1. The ratios of Mix 2 were given as $1 : 2 : 1/2$, but by multiplying through by 2 we can clear it of fractions.

Now we just add up all the fractions of apple juice contributed from each Mix (rate $\times$ quantity):

$$\left(\frac{1}{2}\right)\left(\frac{1}{4}\right) + \left(\frac{2}{7}\right)\left(\frac{11}{20}\right) + \left(\frac{0}{1}\right)\left(\frac{4}{20}\right) = \frac{79}{280}\,. \tag{1}$$

This is roughly .282 is decimal form, corresponding to about 28.2%.

4 Word Problem #27.2

[2] A man has 10 gallons of a 50% sulphuric acid solution, 20 gallons of a 20% solution, and 5 gallons of a 10% solution. He wants to use up all the 10% solution and make 15 gallons of 30% solution. How much of each solution should he use?

5 Solution 27.2.1: Conceptualizing the Problem

Let's begin with a diagram to represent the situation.

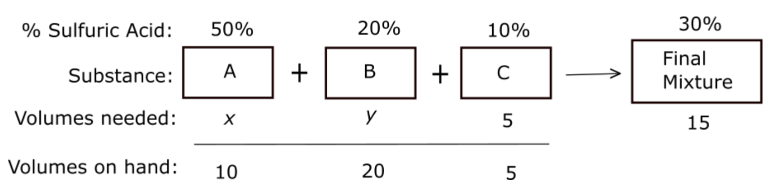

Figure 2. A place for everything and everything in its place. What to do with the information about how much of each ingredient we have on hand? We should include it in the diagram; after all, we will need to test the values of x and y to make sure they do not violate the constraints.

Balancing on overall volumes, we get

$$x + y + 5 = 15\,, \tag{2a}$$

[2]Found at http://johnrdixonbooks.com/images/Word.pdf, p. 6.

which simplifies to

$$x + y = 10\,. \tag{2b}$$

Next, balancing on volumes of sulfuric acid, we get

$$.5x + .2y + .1(5) = .3(15)\,, \tag{3a}$$

which simplifies to

$$5x + 2y = 40\,. \tag{3b}$$

Equations (2b) and (3b) have a solution

$$x = 6\,\frac{2}{3}\,\text{gal.} \quad \text{and} \quad y = 3\,\frac{1}{3}\,\text{gal.} \tag{4}$$

Checking back with the figure, we confirm that these solutions do indeed comply with the amounts on hand of substances A and B.

6 Word Problem #27.3

[3] An employer has a daily payroll of $1950 when employing some workers at $120 per day [type B worker] and others at $150 per day [type A worker]. When the number of $120 workers is increased by 50% and the number of $150 workers is decreased by 1/5, the new daily payroll is $2,400. Find how many workers were originally employed at each rate.

7 Solution 27.3.1: Conceptualizing the Problem

This is a fairly simple mixed-rate problem. Let's start with a figure.

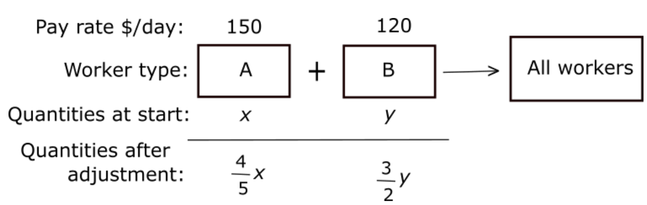

Figure 3. Standard mixed-rate problem. To increase by 50% is to multiply by 3/2. To decrease by a 1/5th is to leave 4/5th.

For each configuration of workers, we have a distinct 'total as the sum of its parts' to write down:

$$150x + 120y = 1950\,, \tag{5a}$$

and

$$150(\tfrac{4}{5}x) + 120(\tfrac{3}{2}y) = 2400\,. \tag{5b}$$

The solution to this system is $x = 5$ and $y = 10$.

[3] Found at https://answers.yahoo.com/question/index?qid=20080214222603AAcQyvt

8 Word Problem #27.4

[4] Two large and 1 small pumps can fill a swimming pool in 4 hours. One large and 3 small pumps can also fill the same swimming pool in 4 hours. How many hours will it take 4 large and 4 small pumps to fill the swimming pool.(We assume that all large pumps are similar and all small pumps are also similar.)

9 Solution 27.4.1: Conceptualizing the Problem

Let $\overline{v}$ stand for average speed and T stand for the total time of the hike.

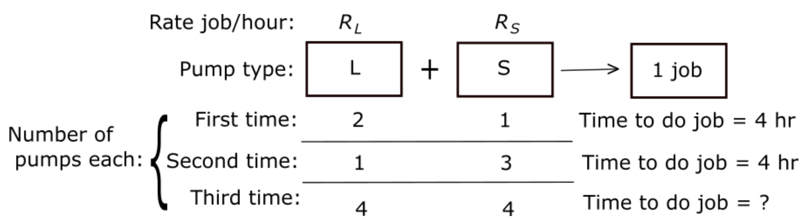

Figure 4. What will be the final time to fill the pool? (Each pump contributes to the total job being done.)

In the first scenario we get

$$(2R_L + R_S)(4\,\text{hrs}) = 1\,\text{job}\,. \tag{6}$$

In the first scenario we get

$$(R_L + 3R_S)(4\,\text{hrs}) = 1\,\text{job}\,. \tag{7}$$

The solution to these last two equations are $R_L = 1/10$ job / hour, and $R_S = 1/20$ job / hour. To determine the time it will take to fill the pool in the third scenario, we need to solve for T in

$$(4R_L + 4R_S)T = (4\frac{1}{10} + 4\frac{1}{20})T = 1\,\text{job}\,. \tag{8}$$

The solution for T in this last equation is $T = 1$ hour 40 minutes.

10 Conclusion

I hope the reader can see how that the graphical represention of the data is better than to employ a tabular form to represent it.

[4]Found at http://www.analyzemath.com/high_school_math/grade_12/problems.html.