Word Problems 28

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Abstract

In this algebra word problem note, we use the Scheme to solve a variety of problem that may or may not include 'mixed-rate' types.

1 Introduction

I just found out that there is a word that denotes the study of part to whole: Mereology

2 Word Problem #28.1

[1] Question 10300: Mary bought some donuts. She gave 1/2 her donuts and 1/2 a donut to her mom. Then she gave away 1/2 her remaining donuts and 1/2 a donut to her aunt. Then she gave 1/2 of her remaining donuts and 1/2 a donut to her sister, Kathy. This left her with 1/4 of a dozen donuts. How many doughnuts had she bought? [Ans. 31]

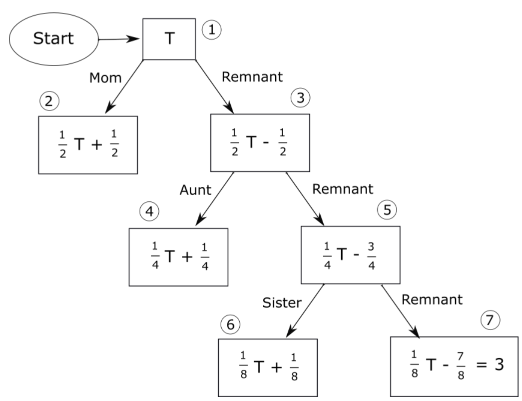

Figure 1. The donut tree.

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[1]Found at https://www.algebra.com/algebra.

3 Solution 28.1.1: Conceptualizing the Problem

My first attempt at solving this problem was to create a lot of temporary variables to represent the given information in a step-wise fashion, but on reconsideration, I think a tree structure diagram is a better heuristic.

Let's look at the graphic in Figure 1. We start with an unknown amount of T donuts and use the given information to work our way down the tree structure. At each level below the top level, we first compute the amount given to a relative and then the remnant is given by

$$\text{Remnant} = (\text{Amount in previous node}) - (\text{Amount given to relative})\,. \quad (1)$$

For example: To compute the first remnant at Point 3, we first compute the amount given to Mom, which is $\frac{1}{2}T + \frac{1}{2}$, and then subtract that amount from the previous node in the tree, which is

$$\text{Remnant at Point 3} = T - (\tfrac{1}{2}T + \tfrac{1}{2}) = \tfrac{1}{2}T - \tfrac{1}{2}\,. \quad (2)$$

So, we keep giving out donuts to appreciative relatives, computing the remnant as we go, until we run out of relatives. That final remnant is calculated and set equal to three. From the remnant node at Point 7 in the figure, we see that $\frac{1}{8}T - \frac{7}{8} = 3$. Solving for T, we find that Mary started with 31 donuts.

4 Word Problem #28.2

[2] Question 6293: In printing an article of 48,000 words, a printer decides to use two sizes of type. Using the larger type, a printed page contains 1,800 words. Using smaller type, a page contains 2,400 words. The article is allotted 21 full pages in a magazine. How many pages must be in smaller type?

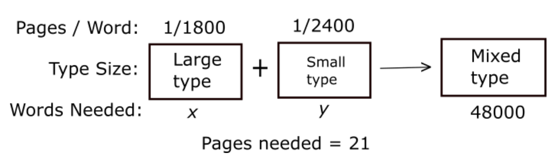

Figure 2. Pages per word? Really? Sure, because that's what works.

5 Solution 28.2.1: Conceptualizing the Problem

In principle, this problem is no different than two machines working together to complete a job. The job here is for these two abstract 'machines' together to consume 21 pages of output. The faster machine gobbles up pages at the rate

[2] Ibid.

of 1/1800 [pages/word], and the slower machine at the rate of 1/2400 [pages/word].

Balancing on total words needed, we get

$$x + y = 48000\,. \tag{3a}$$

Balancing on total pages, we get

$$1800^{-1}x + 2400^{-1}y = 21\,. \tag{3b}$$

The solutions are $x = 7200$ and $y = 40800$.

Thus, the number of pages at the smaller type is (40800 words)$(\frac{1}{2400}\,\frac{\text{pages}}{\text{word}})$ = 17 pages.

6 Word Problem #28.3

Question 2508:[3] Gold is 19 times heavier than water. Copper is 9 times heavier than water. In what ratio should they be mixed so that the alloy is 15 times heavier than water?

7 Solution 28.3.1: Conceptualizing the Problem

This is a fairly simple mixed-rate problem. Let's start with a figure.

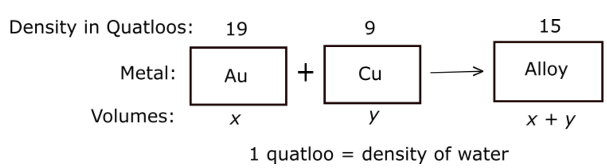

Figure 3. We are tasked with finding the ratio of x to y. In that effort, we don't need to know the actual density of water. Hence, the introduction of the word 'Quatloo' – a metasyntactic variable.

Wikipedia defines a *metasyntactic variable* as a meaningless word used as a placeholder in computer science, but I see no reason not to use them here as well.[4] So I introduced the term *quatloo* as a metasyntactic variable to stand for the density of water, which the problem does not require us to know its actual value to obtain the requested answer. Computer scientists don't use the word 'quatloo'; they use words like 'foo' or 'foobar'. The word 'quatloo' comes from an episode of the original Star Trek series to mean some unknown amount of currency of some unknown alien race on the planet Triskelion.

[3]Ibid.

[4]At least in Scheme, *our* meaningless words can have units.

So why is all this off-topic nonsense of relevance to this problem? First, we've solved so many similar mixed-rate problems that this is the *most* interesting aspect to this problem. Second, before you start to solve any problem, it's important to know just how much of the given information is relevant and in what form the given information is relevant.

In this problem, we needed to know that the units of gold and copper were in densities. But both the densities of gold and copper were given to us in the same way, yet all we need to know in the end is the ratio of gold volume to silver volume, which is a pure number, and can't possibly depend on the density of water. If you need more convincing, imagine recasting the given densities of gold and copper in terms of multiples of the density of aluminum or of liquid nitrogen. It just doesn't matter.[5]

Anyway, our equation for conserving mass across the alloying process is

$$19x + 9y = 15(x + y)\,. \tag{4}$$

The solution to this system is $x : y :: 3 : 2$.

8 Word Problem #28.4

I found this little jem in Blitzer [1]. At the north campus of a small liberal arts college, 10% of the students are women. At the south campus, 50% of the students are women. The campuses are merged into one east campus, of which 40% of the 1200 students are women. How many students were in the north and south campuses before the merger?

9 Solution 28.4.1: Conceptualizing the Problem

Our figure needs to represent all three campuses in the boxes.

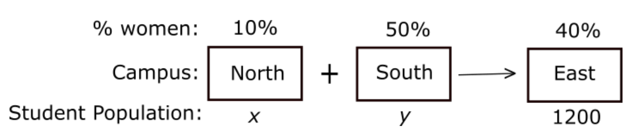

Figure 4. A standard setup.

Balancing on students before and after the merger, we get

$$x + y = 1200\,. \tag{5}$$

Balancing on the number of women before and after the merger, we get

$$.1x + .5y = .4(1200)\,. \tag{6}$$

The solutions to these last two equations are $x = 300$, and $y = 900$.

[5] In fact, introducing the actual density of water (or whatever is being used in its place) could introduce round-off error into the calculation.

10 Word Problem #28.5

[6] A crew is made up of 8 men; the rest are women. $66\frac{2}{3}\%$ of the crew are men. How many people are in the crew?

11 Solution 28.5.1: Conceptualizing the Problem

Although this is a very simple word problem, it never hurts to use Scheme to solve it. We start by drawing a simple picture to represent the basic composition of this crew:

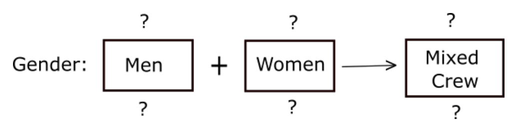

Figure 5. Scheme is a process. We start with the more or less obvious stuff.

Start with the obvious and then figure out where the rest of the given information goes: Above the boxes (rates, percentages, fractional amounts) and below the boxes (quantities) or just information that doesn't fit either of the first two choices. In Figure 6, I added a top line of fractional amounts (converted from the percentages), as it makes the numbers easier to deal with.

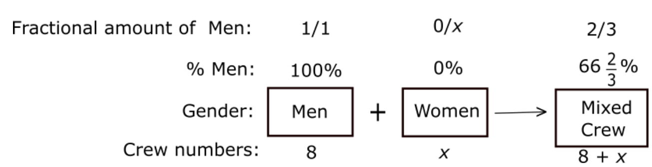

Figure 6. I converted percentages to fractions to facilitate the next part of this process: balancing on some conserved quantity.

Now we balance on the 'men' quantities on both sides:

$$(\frac{1}{1})8 + (0)x = \frac{2}{3}(8 + x)\,. \tag{7}$$

Thus, the total number of people on the crew is $8 + x = 12$.

[6] Found at https://www.cnm.edu/depts/tutoring/tlc/res/accuplacer/8_Math_550_Percent_Word_Problems__2_.pdf.

12 Word Problem #28.6

Question 177914:[7] If a hen and a half can lay an egg and a half in a day and a half, how many eggs will six hens lay in seven days?

13 Solution 28.6.1: Conceptualizing the Problem

When I first read this unusual problem, I had a psychological reaction to it that stopped me for a moment. However, the problem merely poses a total as the result of a conversion factor times two quantities in the form

$$T = R\,Q_1 Q_2\,, \tag{8}$$

where R is the conversion factor (rate of exchange) and the Q's are quantities. Let's set up R first

$$R = \frac{1.5\,\text{eggs}}{(1.5\,\text{hens})(1.5\,\text{days})}\,. \tag{9}$$

Now, let's put it all together:

$$T = \frac{1.5\,\text{eggs}}{(1.5\,\text{hens})(1.5\,\text{days})}\,(6\,\text{hens})(7\,\text{days}) = 28\,\text{eggs}\,. \tag{10}$$

14 Conclusion

This time we learned that metasyntactic variables can be useful in real problem solving, even outside of computer science. Who knew?

We also learned that a hen and a half in a hand and a half is worth two and a half in a bush and a half.

References

[1] R. Blitzer. *Intermediate Algebra for College Students*, 3rd Ed. Prentice-Hall (2002), p. 181 #21.

[7] Found at https://www.algebra.com/algebra.