Word Problems 30

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Abstract

In this algebra word problem note, we use the Scheme to solve a variety of problems. This paper hs more of the same kinds of problems, but we solve one problem using calculus.

1 Introduction

No new surprises. Just more practice.

2 Word Problem #30.1

This problem is from a Khan Academy Youtube video: Advanced Ratio Problem. In a group of 57 children the ratio of girls to boys is $4 : 15$. How many boys must leave the group so that the resulting ratio of girls to boys is $4 : 11$?

3 Solution 30.1.1: Conceptualizing the Problem

We've been given two kinds of algebraic information, This first is that the total number of students in the group is 57, which we set equal to the sum of its parts, B the number of boys and G the number of girls. We are also given a proportion, the stated equality of two ratios.

We start off with the following equations:

$$B + G = 57\,, \tag{1a}$$

and

$$\frac{G}{B} = \frac{4}{15}\,. \tag{1b}$$

After removing x number of boys, we get

$$(B - x) + G = 57 - x\,, \tag{2a}$$

and

$$\frac{G}{B - x} = \frac{4}{11}\,. \tag{2b}$$

Solving (1a) and (1b) together, we find that $B = 45$ and $G = 12$. On substituting those values into (2b) and solving for x, we get

$$x = 12\,. \tag{3}$$

Thus twelve boys must be removed from the group so that the remaning ratio of girls to boys is 4 : 11.

4 Word Problem #30.2

[1] If jogging for 1 mile uses 150 calories and fast walking for 1 mile uses 100 calories, a jogger has to go how many times as far as a walker to use the same number of calories?

5 Solution 30.2.1: Conceptualizing the Problem

One equation jumps out at us: The number of calories used by the jogger = the number of calories used by the walker. Let D_J be the distance the jogger must run to use the same number of calories the walker will use in distance D_W.

Setting up the equation, we get

$$D_J \left[\frac{150 \text{ cal}}{\text{mile}}\right] = D_W \left[\frac{100 \text{ cal}}{\text{mile}}\right]. \tag{4}$$

On simplifying, we get

$$D_J = \frac{2}{3}\, D_W\,. \tag{5}$$

Thus, the jogger needs to go only two-thirds the distance the walker needs to go to burn-up the same number of calories.

6 Word Problem #30.3

Question 123779:[2] A rock is dropped from a cliff into the ocean. It travels $16t^2$ feet in t seconds. If the splash is heard 1.5 second later, how high is the cliff? [Note: Assume the speed of sound at sea level is 1100 feet per second.]

7 Solution 30.3.1: Conceptualizing the Problem

Let h be the height of the cliff and v be the speed of sound. The total time Δt from when the rock is released until the splash is heard is given as the sum

[1] From *Nursing School Entrance Exam*, 2005, LearningExpress, p. 52.
[2] Found at https://www.algebra.com/algebra.

(suppress units)

$$
\begin{aligned}
\Delta t &= \begin{bmatrix} \text{time for rock to} \\ \text{hit the water} \end{bmatrix} + \begin{bmatrix} \text{time for sound of splash} \\ \text{to hit the ears} \end{bmatrix} \\
&= \text{Extract\_Time}[h = 16t^2] + \text{Extract\_Time}[t = h/v] \\
&= \frac{\sqrt{h}}{4} + \frac{h}{1100}\,. \qquad (6)
\end{aligned}
$$

But $\Delta t = 1.5$ seconds. So, letting $z \equiv \sqrt{h}$ and clearing of fractions, we get

$$z^2 + 275z - 1650 = 0\,, \qquad (7)$$

with solution $z \approx 5.8745$. That gives us $h \approx 34.5$ feet.

Thus we see that the Extract_X function inputs an equation and returns the expression or value equivalent to X.