Word Problems 31: One More Hard Problem

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Abstract

In this algebra word problem note, we use the Scheme to solve more difficult word problems.

1 Introduction

All of the word problems in this note are from the website

`https://www.basic-mathematics.com/hard-word-problems-in-algebra.html`

2 Word Problem #31.1

Problem 8. One ounce of solution X contains only ingredients a and b in a ratio of 2:3. One ounce of solution Y contains only ingredients a and b in a ratio of 1:2. If solution Z is created by mixing solutions X and Y in a ratio of 3:11, then 2520 ounces of solution Z contains how many ounces of a? [Also, find the ratio of a to b in Z.]

Solution

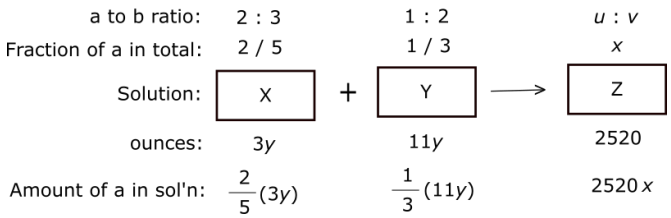

Figure 1. We don't know yet what the value of y is, but when X and Y are mixed together, the result will be in ratio $3y : 11y = 3 : 11$.

First, the information about the number of ounces of X and Y is irrelevant, so we will ignore it. Second, we make a diagram (above). Third, the solving order is first y, then x, then u and v.

We can easily solve for y by utilizing the conservation of ounces in the mixing:

$$3y + 11y = 2520\,, \tag{1}$$

which give us $y = 180$.

Now, utilizing the conservation of ounces of substance a in the mixing, we get:

$$\frac{2}{5}(3y) + \frac{1}{3}(11y) = 2520x\,, \tag{2}$$

or

$$x = \frac{180}{2520}(\frac{6}{5} + \frac{11}{3}) = \frac{73}{210}\,. \tag{3}$$

Therefore, the number of ounces of a in Z is

$$2520x = 2520\frac{73}{210} = 876\,. \tag{4}$$

Lastly, we're asked to find the ratio of a to b in Z, which is represented by the ratio $u : v$ in the diagram. But,

$$x = \frac{u}{u+v} = \frac{73}{210}\,. \tag{5}$$

Now, if we let $\lambda = v/u$, then this last equation becomes

$$x = \frac{1}{1+\lambda} = \frac{73}{210}\,. \tag{6}$$

Solving this for λ yields

$$\lambda = \frac{137}{73}\,, \tag{7}$$

from which we get that

$$u : v = 73 : 137\,. \tag{8}$$

3 Word Problem #31.2

We can do this easily. The total heat lost for both walls is equal to the sum of the heats lost through their glass parts and their plaster parts:

$$\begin{aligned} 1920 &= 40x + 60y\,, \\ 1160 &= 10x + 100y\,, \end{aligned} \tag{9}$$

where $x = R_G$ and $y = R_P$. This makes it easier to copy the text into the solver, which gives back $x = R_G = 36$ [BTU] and $y = R_P = 8$ [BTU].

4 Conclusion

To solve tricky problems, it helps to know the available tricks.

References

[1] R. Blitzer. *Intermediate Algebra for College Students*, 3rd Ed. Prentice-Hall (2002), p. 169.

[2] H. Rolf. *Finite Mathematics*, 5th Ed. Brooks/Cole (2002), p. 57.