Word Problem 3: Using algebra on an unbalanced chemical equation

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Abstract

In this algebra word problem, we use the Scheme to solve another chemistry problem.

1 PROBLEM THREE

Now we're going to use the above-mentioned rules to find a method of solving for the coefficients to unbalanced chemical equations algebraically. I intend to show that the rules and heuristics already given in this paper will push us to the solution at practically the speed of thought.

Balance the following unbalance chemical equation:

$$\mathrm{KCN} + \mathrm{FeCl_2} \to \mathrm{K_4Fe(CN)_6} + \mathrm{KCl}\,. \tag{1}$$

2 Solution Part 1: Conceptualizing the problem

Now, according to RULE ONE we have to give each unknown coefficient a symbol. So, (1) becomes

$$x_1\mathrm{KCN} + x_2\mathrm{FeCl_2} = x_3\mathrm{K_4Fe(CN)_6} + x_4\mathrm{KCl}\,. \tag{2}$$

Thus it appears we have four unknowns to solve for, but if we think a moment, we'd see that we only need to solve for three unknowns since the coefficients are only determined up to their mutual ratios.

By RULE TWO, we know that we need three conditions from the equation itself, and the fourth condition we set ourselves by arbitrarily choosing $x_4 = 1$, say. Thus (2) becomes

$$x_1\mathrm{KCN} + x_2\mathrm{FeCl_2} = x_3\mathrm{K_4Fe(CN)_6} + \mathrm{KCl}\,. \tag{3}$$

According to RULE FOUR we need to decide what the goal of the problem is. It is to balance the equation. But what does that really entail? It means that we need a solution for the coefficients that will balance all the elements at the same time.

Now, we can think of this as a conservational problem: The total number of each element is conserved going from the left-hand side to the right-hand side.[1] Bingo. There's that word "total" again. Every total is equal to the sum of its parts. What are the parts, then? The parts are the contributions of the particular element from each term. Therefore we can write the conservational equation for Potassium, K, yielding

$$\text{Total K on LHS} = \text{Total K on RHS} \tag{4}$$

and the two others we need follow similarly. Then summing up and equating the term-wise contributions for K, N, and Fe gives

$$\begin{array}{lcccccccc} \text{K} & : & 1x_1 & + & 0x_2 & = & 4x_3 & + & 1 \\ \text{N} & : & 1x_1 & + & 0x_2 & = & 6x_3 & + & 0 \\ \text{Fe} & : & 0x_1 & + & 1x_2 & = & 1x_3 & + & 0 \end{array} \tag{5}$$

There are a number of ways to solve (5). One way is to subtract the first equation from the second, yielding $x_3 = \frac{1}{2}$. Substituting this value into the second gives $x_1 = 3$. And from the third equation we get $x_2 = x_3 = \frac{1}{2}$. Thus (3) becomes

$$3\text{KCN} + \tfrac{1}{2}\text{FeCl}_2 = \tfrac{1}{2}\text{K}_4\text{Fe(CN)}_6 + \text{KCl}\,. \tag{6}$$

On multiplying this through by 2 we get

$$6\text{KCN} + \text{FeCl}_2 = \text{K}_4\text{Fe(CN)}_6 + 2\text{KCl}\,. \tag{7}$$

We got coefficients, sure enough, but we're not finished yet. According to the goal statement, we still need to verify that the coefficients work for C and Cl, which they do.

We're still not finished because we need to solve the general problem, not just our test case. One thing we learned is that an equation with n terms needs only $n-1$ elements to solve for $n-1$ coefficients. But what happens if we encounter a chemical reaction that has fewer than $n-1$ elements? The answer is that we can't solve for the coefficients uniquely. Therefore we invoke the Zeroth Rule to require that all reactions dealt with by this algorithm will involve at least $n-1$ elements for n terms.

3 Conclusion

I haven't done much with graphics to help clarify the structure of the word problems yet. However, graphics are so important conceptually that they're worth my time to make them and the reader's time to study them. I think this will be evident as we progress in this series of papers

[1] In saying 'number', we can think in terms of individual atoms or in terms of moles.