Word Problems 4: The Mixed-Rate Problems #1

P. Reany

March 22, 2019

Abstract

In this algebra word problem note, we use Scheme to solve our first attempt at what I refer to as a 'mixed-rate problem'. In this type of problem, two or more 'machines' work together at generally different rates and different time intervals to produce subtotals that add to a total.

1 Introduction

We begin this kind of problem by considering an actual machine pair working together to accomplish a job, and generally they work at different rates. If the machines start and stop at the same time during the job, the problem is relatively simple. If they don't, the problem is a bit more difficult. Remember, in this kind of problem, do not rush to identify 'key variables'. Instead, look for totals and parts; the appropriate variables will fall out of the ensuing analysis.

For our first two examples of this kind of problem, I will demonstrate problems where the notion of machine is literal. The first being a printer and the second being an oil pump.

2 Word Problem #4.1

Consider the following problem: Printer #1 can print a 100 copies of a document in 3.4 hours and Printer #2 can print out the same print job in 2.5 hours. How long will it take for the print job to complete if both printers work on the job together, starting and stopping at the same time?

3 Solution Part 4.1.1: Conceptualizing the problem

We introduce the shorthand 'part of job done by' → PJDB. Then our highest-level equation is

$$1 \text{ job} = (\text{PJDB Printer } 1) + (\text{PJDB Printer } 2) . \tag{1}$$

Let R_1 be the average rate at which Printer 1 can work, which is 1 job/3.4 hours. Likewise, R_2 is the average rate at which Printer 2 can work, which is 1 job/2.5 hours. Now, the most general expression we can write for the refinement of the last equation is[1]

$$1 \text{ job} = R_1 T_1 + R_2 T_2 \,, \tag{2}$$

where T_1 and T_2 are the respective times that Printer 1 and Printer 2 are operating. For the current problem, these two times are equal and they are equal to the total time the print job takes, but the last equation is the most general for two printers. So, for our current problem, let's set this common time equal to T and suppress units, to get

$$1 = R_1 T + R_2 T = (R_1 + R_2) T \,. \tag{3}$$

Perhaps this equation is beginning to look famliar to you from your previous algebra experience (such as in the problems given in the SAT, GRE, or LSAT). Solving for T, we get

$$T = \frac{1}{R_1 + R_2} \,. \tag{4}$$

If this equation looks close to what you remember, but not quite right, that could be because you're used to thinking of the rates in units of [hours/job], the inverse of the units for R. So, letting $G_1 = R_1^{-1}$ and $G_2 = R_2^{-1}$, we get[2]

$$T = \frac{1}{G_1^{-1} + G_2^{-1}} = \frac{G_1 G_2}{G_1 + G_2} \,. \tag{5}$$

4 Solution Part 4.1.2: Solving the problem

Anyway, using the given values of R_1 and R_2 plugged into (4), we get $T = 1.44$ hours $\approx$ 1 hour 26 minutes.

The first question we should ask of this answer is if it's reasonable, or as they say, is it 'in the ball park'. For starters, it's less than the shorter individual time, so that's a good sign. Second, what answer would we get if the problem were changed to two printers working at 3 hours each (that value is roughly the average of the two job times)? Both working together should give half the time of either working alone, which is 1.5 hours. And this answer is close to 1.44 hours.

Let's change the problem slightly to make a point. The boss sends me an email telling me that the print job isn't due till next week, and that I can have Printer 1 all day Wednesday and Printer 2 all day Thursday. Is that OK? I email back that's fine. Here's why: I just run Printer 1 for the scheduled job for 1.44 hours on Wednesday and run Printer 2 for its scheduled job for 1.44 hours on Thursday. The print job is finish on Thursday, even though the two printers did not run at the same time.

[1] We are employing the Zeroth Rule of Problem Solving to make the simplifying assumption that the average rate will be accurate for arbitrarily long or short time intervals.

[2] By my choosing G for the inverse of R, I have intensionally made a formal analogy between the variables here and resistence and conductance in electric-circuit analysis.

5 Word Problem #4.2

The main oil pump at an oil refinery can fill a tanker in 2 hours. The engineer in charge is concerned that the main pump is in need of repair and he doesn't want to stress it too much. He has a pumping window of 4 hours to fill a tanker before the main pump goes off-line. He also has available a slower pump that can fill the tanker in 6 hours. If the engineer wants to start the tanker job with the slower pump and then add in the main pump at the last possible moment, and then run both simultaneously until the tanker is full, how long will the main pump be used?

6 Solution Part 4.2.1: Conceptualizing the problem

Let's refer to the faster pump (the main pump) as Pump 1, and the other pump as Pump 2. We have 1 job to be accomplish, split into two contributions:

$$1 \text{ [job]} = (\text{PJDB Pump 1}) + (\text{PJDB Pump 2})\,. \tag{6}$$

And the next refinement gives us (suppressing units)

$$1 = R_1 T_1 + R_2 T_2\,, \tag{7}$$

where T_1 is the time the main pump will run, and T_2 is the time the slower pump will run, which we know will be the full 4 hours. Also, we know that $R_1 = \frac{1}{2}$[job/hour] is the rate at which the main pump will run, and $R_2 = \frac{1}{6}$[job/hour] is the rate at which the slower pump will operate. Substituting these values into the last equation, we get that

$$1 = \frac{1}{2}\,T_1 + \frac{1}{6}\,4\,. \tag{8}$$

7 Solution Part 4.2.2: Solving the problem

Solving this for T_1 we get,

$$T_1 = \frac{2}{3}\,\text{hour} = 40 \text{ minutes}\,. \tag{9}$$

This means that the engineer needs to turn on the main pump after the slower pump has been running for 3 hours and 20 minutes.

8 Conclusion

In the future mixed-rated problems presented in this series of notes, the notion of a 'machine' will become very abstract: It may be two painters working together on a single paint job. It could be two people mowing the same lawn. It could be two blocks of text at different point sizes ‘working’ to fill a single page esthetically. We’ll also encounter problems where there will be more than just two ‘machines’ working at the same time, or at least on the same job.