Word Problems 8: The Mixed-Rate Problems #5

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Abstract

In this algebra word problem note, we use the Scheme to solve our fifth attempt at what I refer to as a 'mixed-rate problem'. In this type of problem, two or more 'machines' work together at generally different rates to produce subtotals that add to a total. Quantitative information can be given in the problem in various forms: percentages, fractional amounts, or by ratios. We have to know how to deal with each of them.

1 Introduction

This time we do four problems. Problem 3 converts between ratios and fractions.

2 Word Problem #8.1

In what ratio should water be added to a liquid costing \$12 per liter so as to make a profit of 25% by selling the diluted liquid at \$13.75 per liter?

3 Solution 8.1.1: Conceptualizing the problem

We'll worry about the ratio after we have calculated how much water should be added to the starting liquid, which we'll set at 1 liter. We lose no generality by doing this.

But first, a word about this 25% profit. How do we deal with it? Percentage changes to a decimal 0.25 as a multiplier. I won't go into detail because I offer this only as a refresher to what the reader is presumed already familiar with.

$$\begin{aligned}
(\text{retail cost}) &= (\text{base cost}) + (\text{profit}) \\
&= (\text{base cost}) + (0.25)(\text{base cost}) \\
&= (1.25)(\text{base cost})
\end{aligned}$$

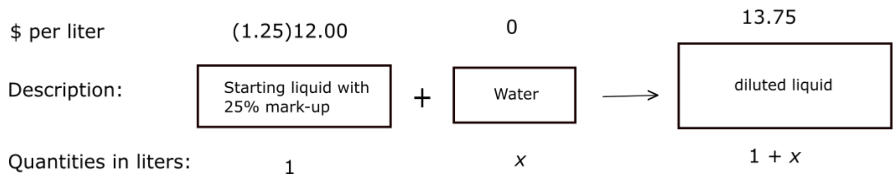

Figure 1. This graphic represents the adding some water x to a starting liquid in a 'before and after' process. The arbitrary markup has already been applied to the cost per liter before adding water.

In the graphic in Figure 1, we show a 'before and after' process of adding water to this starting liquid. We begin with the conservation equation

$$\text{(markedup cost of original liquids)} = \text{(required cost of diluted liquid)}\,. \tag{1}$$

The next adjustment is arbitrary: We are told that the marked-up cost to the consumer will bring in a 25% profit. In other words, the retail cost of the diluted liter of liquid is (1.25)($12.00). But the retail cost of the diluted liter of liquid is also given as ($13.75)$(1 + x)$, from which we get the equation (in dollars)

$$(1.25)(12.00) + 0 \cdot x = (13.75)(1 + x)\,. \tag{2}$$

Solving this, $x = 0.090909....$ But we are asked to find the ratio of $x : 1$, which is $0.090909 : 1$, or (approximately) $1 : 11$.

4 Word Problem #8.2

A merchant has 100 lbs of sugar, part of which (x lbs) he sells at 7% profit and the rest (y lbs) at 17% profit. The division of the whole into two parts is to be made so that the net profit is the same as 10% on each original quantity of sugar. How much is each part?

5 Solution 8.2.1: Conceptualizing the problem

Let's begin with a figure to help us conceptualize the data.

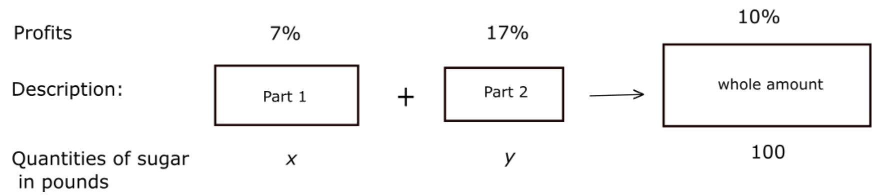

Figure 2. How to divide 100 pounds of suger to get 10% profit.

First, it might seem that we have the arrow backward in Figure 2. Shouldn't we start with the 100 lbs and then divide it up into parts? Well, we will eventually make that division into x and y parts, but until we solve the conceptual problem posed in the Figure 2, we don't know what those parts are. The conceptual problem looks at it the other way: The net profit of 10% results from the right choice of x and y, yielding the correct subtotals that add to a total.

So, we have two equations in two unknowns, beginning with the conservation of weight of sugar (in pounds):

$$x + y = 100\,. \tag{3}$$

And we have the conservation of profit:

$$(\text{profit off of } x) + (\text{profit off of } y) = (\text{net profit off of 100 lbs})\,. \tag{4}$$

For the next refinement, we'll convert percentages to decimals and multiply rates times quanties off Figure 2, to get

$$.07x + .17y = .10 \cdot 100 = 10.00\,. \tag{5}$$

Solving (3) and (5) together yields $x = 70$ and $y = 30$ in pounds.

6 Word Problem #8.3

Two vessels A and B containing milk and water in ratios $4 : 3$ and $2 : 3$, respectively. In what ratio should they be added together so that their final mixture is in ratio $1 : 1$?

7 Solution 8.3.1: Conceptualizing the problem

Notice in Figure 3 that we used arbitrary volume units. One reason for this is that we weren't given a specific unit to work with, and the other is that we can choose any particular unit we please because in taking ratios the units will cancel.

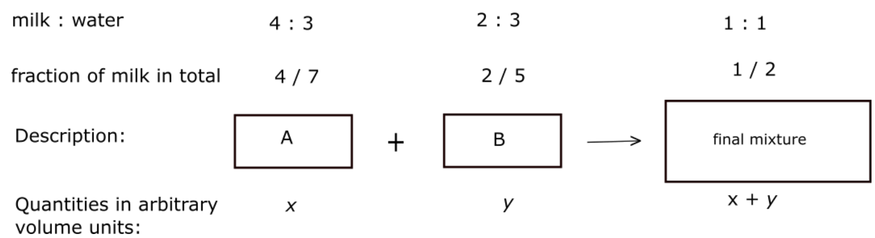

Figure 3. We need to solve for the ratio of x and y.

Now, we've already shown the conservation of volume in the bottom line. We need now only one more equation in x, y to solve for their ratios (hopefully). For that, we show the conservation of milk on both sides.[1]

$$(\text{milk in } A) + (\text{milk in } B) = (\text{milk in final mixture})\,. \tag{6}$$

8 Solution 8.3.2: Solving the problem

From (6) we get

$$\frac{4}{7}x + \frac{2}{5}y = \frac{1}{2}(x+y)\,. \tag{7}$$

The variable we need to solve for is x/y, and to do this efficiently, let's divide the last equation through by y, to get

$$\frac{4}{7}x/y + \frac{2}{5} = \frac{1}{2}(x/y+1)\,. \tag{8}$$

Let's make one more simplification and substitute $\lambda = x/y$ to get

$$\frac{4}{7}\lambda + \frac{2}{5} = \frac{1}{2}(\lambda+1)\,, \tag{9}$$

with solution $\lambda = 21/5$. Therefore $x : y :: 21 : 5$.

9 Word Problem #8.4

A can contains a mixture of two liquids A and B in ratio $7 : 5$. After 9 liters are drawn off and replaced by 9 liters of liquid B, the ratio of A to B becomes $7 : 9$. How many liters of liquid A was in the can initially.

[1] The figure was setup to show the conservation of milk, but we could just as easily have shown the conservation of water.

10 Solution Part 8.4.1: Conceptualizing the problem

Let x represent the original total liquid contents of the can. Since we draw off 9 liters and replace it by 9 liters, the final liquid will have x liters in it. Once we determine x, we can then solve for the initial value of A in the can. To simplify the analysis, we'll take as our 'effective' starting condition the state just after the 9 liters of fluid has been drawn off the can.

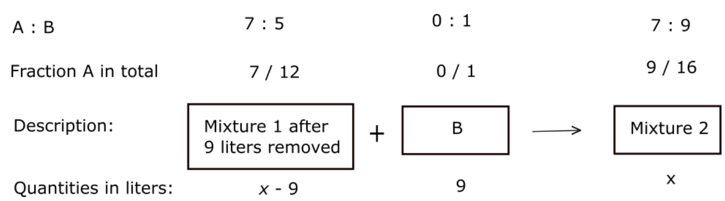

Figure 4. This graphic represents the adding some water x to a starting liquid in a 'before and after' process. 'Fraction A in total' means A/(A+B).

We begin with our usual conservation equation, this time for A.

$$(\text{total } A \text{ in can before adding } B) = (\text{total } A \text{ in can after adding } B)\,. \tag{10}$$

Putting in some details, we get

$$\frac{7}{12}(x-9) + 0 \cdot 9 = \frac{9}{16}(x)\,. \tag{11}$$

11 Solution Part 8.4.2: Solving the problem

Therefore the solution for x is 252 liters. And since A was seven-twelfths of x,

$$\text{inital amount of } A = \frac{7}{12} \cdot 252\,\text{liters} = 147\,\text{liters}\,. \tag{12}$$

12 Conclusion

In problems requiring only the ratio of two variables with the same unit, one needs less information, namely the unit can be dropped. Also, using a 4-layer figure may make accommodating ratio data easier to deal with.