Word Problems 9: The Mixed-Rate Problems #6

P. Reany

November 28, 2016

Abstract

In this algebra word problem note, we use the Scheme to solve our sixth attempt at what I refer to as a 'mixed-rate problem'. In this type of problem, two or more 'machines' work together at generally different rates to produce subtotals that add to a total. Quantitative information can be given in the problem in various forms: percentages, fractional amounts, or by ratios. We have to know how to deal with each of them.

1 Introduction

In the first of our problems, our 'machines' are how two different concentrations of water-paint mixtures are combined to produce a third water-paint mixture. In the second problem, we need to determine how to modify a gas-to-oil mixture.

2 Word Problem #9.1

An inlet pipe X can fill a tank in 10 hours. Another, Y, can fill it in 15 hours. An outlet drain Z can empty the tank in 20 hours. If we start with an empty tank and then pipe X is turned on and left on until te tank is full, and then an hour after X is started, pipe Y is also turned on, and then an hour after that, drain Z is opened, and all continue in their run modes until the tank is full. How long will that take?

3 Solution 9.1.1: Conceptualizing the Problem

If you think about it, this problem is not much different than the ones we've encountered before, only this time, we're dealing with three, instead of two, machines working together. And just like previous problems, here we have a total of one job being completed by the contributions of cooperating/competing machines, each providing their own subtotal to the total. And as before, we must calculate each subtotal by a product of a rate times a quantity of time of operation.

Let's begin by defining T as the time to fill the tank, starting when X begins its operation. This is also the quantity of time that X runs. Y runs an hour less than that, $T - 1$. And Z runs an hour less than Y, which is $T - 2$.

In the figure below, we show all three 'machines' contributing their parts of the job to get 1 job completed, though the drain gets a minus sign for competing against the goals of the two inlets.

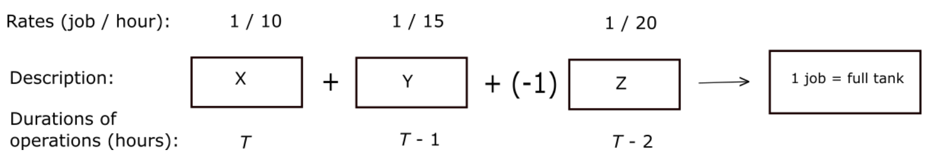

Figure 1. This graphic represents the three machine working to fill a tank.

We begin our analysis by considering that this is a 'total= sum of its parts' problem. The parts are the contributions (subtotals) contributed by the three 'machines', the drain working against the efforts of the two inlet machines, and thus gets a minus sign in front of it

$$1(\text{job}) = (\frac{1}{10}\ \text{job/hour})T + (\frac{1}{15}\text{job/hour})(T-1) - (\frac{1}{20}\text{job/hour})(T-2)\,. \quad (1)$$

Before we solve this equation for T, let's get an estimate first. Let's first estimate how long it would take for just the two inlets to fill the tank if we replaced the ones given by two inlets operating at their everage rate of 25/2 hours/job. And since they are working together, they should take half the time of one of them, that being 25/4 hours/job, which is approximately 6 hours. And the result of operating the drain would be to slow this process down a bit, maybe by a couple hours. Let's see what the algebra says.

This equation has solution for T as

$$T = 58/7\,\text{hours}\,, \quad (2)$$

or 8 hours and a little over 17 minutes, as the crow flies.

By the way, I let wolframalpha.com solve for T by inputting into its solver the line

```
solve[T,1=\frac{1}{10}(T)+\frac{1}{15}(T-1)-\frac{1}{20}(T-2)]
```

which, incidentally, I printed in Latex using the *verbatim* mode.

4 Word Problem #9.2

Judy wants to change the gas-oil mix in her chainsaw from 32:1 to 40:1. If the chainsaw currently has 2 gallons of mixture in it, how much pure gas must be added? (Approximate your answer.)

5 Solution 9.2.1: Conceptualizing the Problem

Let's begin with a figure to help us conceptualize the data.

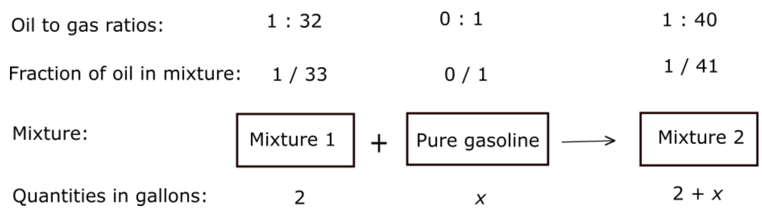

Figure 2. Sometimes the the bridge between a mistake and its fix is just to do the math.

In this before-and-after process, we have already indicated the presumed conservation of volumes. We now need only one equation in x to finish. We can choose to write down the conservation equation for either oil or for gasoline. Let's choose oil, for the simpler equation to solve.

$$\frac{1}{33}\,2 + \frac{0}{1}\,x = \frac{1}{41}(x+2)\,, \tag{3}$$

with solution $x = 16/33$ gallons, or approximately 1/2 gallon.

6 Word Problem #9.3

Starting with 100 lbs alloy of 20% copper and 5% tin, how many pounds of copper and pounds of tin must be melted into it to produce a new alloy that's 30% copper and 10% tin?

7 Solution 9.3.1: Conceptualizing the Problem

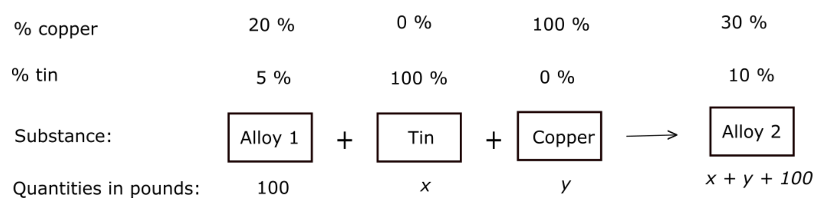

Figure 3. This graphic represents the adding three things together, instead of the usual two things. Reasonably, pure tin has no copper in it, and pure copper has no tin in it.

In this before-and-after process, we will represent the amount of tin to be added as x and the amount of copper to be added as y. In Figure 3, we can see that our convidence that the weights of the constituent parts is preserved.

We have two unknowns to solve for, so we need two coupled equations to solve for them.

$$\text{Conservation of Copper:} \quad .2(100) + 0(x) + 1.0(y) = .3(x + y + 100)\,, \tag{4a}$$

$$\text{Conservation of Tin:} \quad .05(100) + 1.0(x) + 0(y) = .1(x + y + 100)\,. \tag{4b}$$

8 Solution 9.3.2: Solving the Problem

This last pair of equations simply to

$$-.3x + .7y = 10\,, \tag{5a}$$

$$.9x - .1y = 5\,. \tag{5b}$$

When I input the data:

```
[-.3x +   .7 y = 10, .9x -   .1y =5]
```

into wolframalpha.com, I got back the solutions $x \approx 7.5$ and $y \approx 17.5$.

9 Word Problem #9.4

Pipe A can fill a pool 1.25 times faster than Pipe B. If it takes 5 hours when both pipes work together to fill the pool, how long would it take the slow pipe to fill the pool by itself?

10 Solution 9.4.1: Conceptualizing the Problem

Let x represent the time taken by the fast pipe to fill the pool, then

$$R_F = \frac{1}{x}\left[\frac{\text{job}}{\text{hours}}\right] \quad \text{and} \quad R_S = \frac{1}{1.25x}\left[\frac{\text{job}}{\text{hours}}\right]. \tag{6}$$

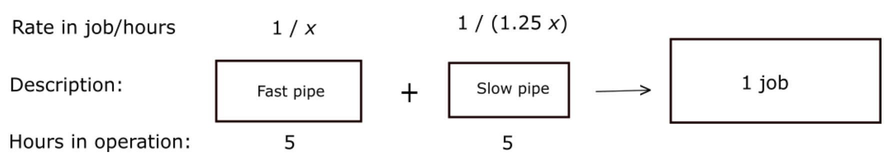

Figure 4. This graphic represents the adding some water x to a starting liquid in a 'before and after' process.

So, the total of 1 job being done is the sum of it parts:

$$1[\text{job}] = \frac{1}{x}\cdot 5 + \frac{1}{1.25x}\cdot 5\,. \tag{7}$$

This equation as solution $x = 9$ hours. Therefore, the slower pipe can fill the pool in $1.25 \cdot 9 = 11.25$ hours.

11 Conclusion

The beauty of the problems presented here is that they are so real-world. For example, if you mix your oil and gas incorrectly for your company power tool, don't panic. So long as you know the ratio you mixed them by, there is an algebraic solution. You don't have to use the wrong mix or merely guess how to fix it. Just do the math!