My Defence of Structured Differentiation from 1999, 1
(or, H. Potter and the Voldemort Derivative)

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Abstract

Here I review my defence of Structured Differentiation which I had made in 1999 on sci.math.

1 Introduction

In 1999, I made a defence on sci.math of my notation in Structured Differentiation (SD), which is a notational system I invented to deal with the many confusing (and well-recognized) features that commonly arise in multi-variable calculus. A mathematician on the newsgroup thought he should counter my claims and I'll present his arguments, and my counterarguments. The reader can decide the merits of my system for him or herself.

I think it will become obvious to the reader that the reason partial derivatives is a confusing subject is simply because it employs too few symbols to chase too many concepts. All SD does is to add in a couple more symbols to better distribute the congitive workload.

Of all the mathematics subjects I've published on in the AJNP the most controversial one is what I call *Structured Differentiation* (SD), which reorganizes and reformulates the so-called theory of "partial differentiation." "Defender" is an alias for a mathematician that defended the status quo for doing so-called partial differentiation [as it was commonly accepted at that time] against my presentation of SD (I have interjected "editorial" comments within square brackets.):

2 My first reply to Defender (15 November)

Subject: Re: partial derivation
Date: Mon, 15 Nov 1999 22:28:37
From: Patrick Reany
Newsgroups: sci.math

Defender wrote:

> In article <382EE601.C75EAA61> Patrick Reany
> <reany> writes:
>
>> You are right to be concerned because the subject [partial
>> derivatives] has a well-deserved notorious reputation.
>
> I don't know if it has a notorious reputation, but if it does, it is
> certainly NOT well-deserved.

I suppose calculus instructors are likely not to see partial differentiation as difficult, but the majority of students I've talked to since 1982 who have had to deal with them do. I have only seen one book that deals with 'partial differentiation' treat them as generalized total derivatives, and that was a Mathematica book! Notations commonly used for partial derivatives are clumsy, overloaded, and presented as a thing apart from ordinary derivatives, all of which adds to their well-deserved notorious reputation – that is, as they are conventionally presented to students.

In his *Advanced Calculus* book (3rd ed), Buck points out that the del sign [∂] is overloaded because it is used for both a total and explicit derivative (eq 3-20, pp137–8). He explains that the partial derivative must be understood in the 'correct context.' But SD always makes this context obvious, and besides, there is no overloading of the del sign in SD to mean more than one possible operation.

As another example of the problems that can occur by this particular operator overloading, consider the case found in Taylor-Mann (*Advanced Calculus*, 2nd ed, p 271): "Consider the function $G(x, y)$ as a function of u and y, with $x = f(u, y)$. The partial derivative with respect to y is

$$\frac{\partial G}{\partial x}\frac{\partial f}{\partial y} + \frac{\partial G}{\partial y} = \text{etc.''} \tag{1}$$

The strange thing with this is the fact that the authors say they are taking a 'partial derivative' but don't say what they are taking it of.

Thus we encouter this ghost derivative, or rather, the Voldemort derivative whose name and symbology shall not be manifested.

Perhaps this is because if they said they were taking the 'partial derivative of G with respect to y' they would have gotten instead:

$$\frac{\partial G}{\partial y} = \frac{\partial G}{\partial x}\frac{\partial f}{\partial y} + \frac{\partial G}{\partial y}\,, \tag{2}$$

but then $\partial G/\partial y$ would appear on both sides. Naturally we are asked to interpret it in its 'proper context.' So the beginner is to interpret the LHS as a sort of 'total' partial derivative, while the similar term on the RHS is a 'partial' partial derivative. This is silly, but necessary to keep them separate. The sad truth is that the expression $\partial G/\partial y$ can mean more than one thing in a given problem, which ain't easy for a student to grasp, let alone deal with effectively. (In rhetorical lingo we'd call this the error of 'equivocation.') The SD notation eliminates all this nonsense by substituting the equation:

$$\frac{\delta G}{\delta y} = \frac{\partial G}{\partial x}\frac{\partial f}{\partial y} + \frac{\partial G}{\partial y}\,, \tag{3}$$

where the term on the left is a total derivative and where the first term on the RHS is the copartial and the second the partial derivative. No confusion. No omitted phrases. No overloading of symbols. And no dodging the fact that the main derivative is a Total derivative, just as in ordinary differentiation.

So, advanced calculus textbook authors talk about the shortcomings of the standard notations, but no one does anything to correct the problem significantly. A whole new way of looking at the problem is required.

>
> There are good reasons why it will be scorned. Follow your
> instructor's notation.
>
> Defender

I already advised the poster to follow the instructor's notation. But it's conventional notations for so-called 'partial differentiation' that should be scorned. Many students do, but don't know what alternative they can turn to. [I suppose they expect that they'll just have to 'get used to them', as John von Neumann explained to us.]

Patrick