

Olympiad Problem 146

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Source: The Ether of Great Mathematical Ideas
Presenter: Patrick

1 The Problem

Given the relation

$$\sqrt{x+1} + \sqrt{x-1} = 2, \quad (1)$$

find the value of x .

2 The Solution

So, I chose as my 'first unipode' and follow-up:

$$a = \sqrt{x+1}u_+ + \sqrt{x-1}u_- \quad (2a)$$

$$= \frac{1}{2}[\sqrt{x+1} + \sqrt{x-1}] + \frac{1}{2}[\sqrt{x+1} - \sqrt{x-1}]u \quad (2b)$$

$$= 1 + \frac{1}{2}\beta u, \quad (2c)$$

where $\beta \equiv \sqrt{x+1} - \sqrt{x-1}$. By squaring this, we get

$$a^2 = 1 + \frac{1}{4}\beta^2 + \beta u. \quad (3)$$

On squaring a in (2a), we have that

$$a^2 = (x+1)u_+ + (x-1)u_- = x + u. \quad (4)$$

On comparing a^2 between (3) and (4), we have that

$$x = 1 + \frac{1}{4}\beta^2, \quad \beta = 1, \quad (5)$$

which gives us

$$x = \frac{5}{4}, \quad (6)$$