Olympiad Problem 149

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The ignorant always loud in argument.
— Charlie Chan

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Source: https://www.youtube.com/watch?v=j1zdTAsfbbI
Title: This Radical Equation is EASIER Than it Looks
 Presenter: NonsoMaths
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1 Problem

Given the relation

$$\sqrt{2x^2 - 7x + 1} - \sqrt{2x^2 - 9x + 4} = 1\,, \tag{1}$$

solve for real values of x.

2 Prerequisites

I'll be using the Unipodal algebra to solve this. If you need some background on it, I have a writeup on it. One particular is that $u^2 = 1$. Also,

$$u_+^2 = u_+ \quad \text{and} \quad u_-^2 = u_-\,. \tag{2}$$

3 Solution

I'll take as my first unipode,

$$\begin{aligned} a &= \sqrt{2x^2 - 7x + 1}\,u_+ + \sqrt{2x^2 - 9x + 4}\,u_- && (3a) \\ &= \tfrac{1}{2}[\sqrt{2x^2 - 7x + 1} + \sqrt{2x^2 - 9x + 1}] + \\ &\qquad \tfrac{1}{2}[\sqrt{2x^2 - 7x + 1} - \sqrt{2x^2 - 9x + 4}]\,u && (3b) \\ &= \tfrac{1}{2}\beta + \tfrac{1}{2}u\,, && (3c) \end{aligned}$$

where we used (1) and where

$$\beta \equiv \sqrt{2x^2 - 7x + 1} + \sqrt{2x^2 - 9x + 4}\,. \tag{4}$$

Next, we square a:

$$a^2 = \left(\tfrac{1}{4}\beta^2 + \tfrac{1}{4}\right) + \tfrac{1}{2}\beta u\,. \tag{5}$$

So, let's recap. We got this last form for a^2 by first converting a to standard form and then squaring. Next, we first square a and then convert that to standard form. After that, we'll equate, respectively, the complex and uniplex parts of both forms.

$$\begin{aligned} a^2 &= (2x^2 - 7x + 1)\,u_+ + (2x^2 - 9x + 4)\,u_- \qquad (6)\\ &= \tfrac{1}{2}(4x^2 - 16x + 5) + \tfrac{1}{2}(2x - 3)\,u\,. \qquad (7) \end{aligned}$$

So, we begin by equating the scalar parts, to get

$$\left(\tfrac{1}{4}\beta^2 + \tfrac{1}{4}\right) = \tfrac{1}{2}(4x^2 - 16x + 5)\,, \tag{8}$$

or simplifying a bit

$$\beta^2 + 1 = 2(4x^2 - 16x + 5)\,. \tag{9}$$

And, on equating the unipotent parts, we have that

$$\tfrac{1}{2}\beta = \tfrac{1}{2}(2x - 3)\,. \tag{10}$$

On eliminating β between Equation (9) and (10), we get (eventually)

$$x(x - 5) = 0\,, \tag{11}$$

which has solutions

$$x = 0 \quad \text{and} \quad x = 5\,. \tag{12}$$

Checking both of these possible solutions to (1), we get only that $x = 5$ works.