Olympiad Problem 150

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December 31, 2025

> There is much you have to learn. You must explore; you
> must reach out. Go...and give thought to the
> the mysteries of the universe.
> — The Galaxy Being
> (An early proponent of
> Life-Long Learning)

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Source: https://www.youtube.com/watch?v=QvW1zHap2D4
Title: A nice radical maths olympiad question
 Presenter: Rashel's Classroom
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1 Problem

Given the relation

$$\sqrt{39-x}+\sqrt{7-x}=8\,, \tag{1}$$

solve for real values of x.

2 Solution

I'll take as my first unipode,

$$\begin{aligned} a &= \sqrt{39-x}\,u_+ + \sqrt{7-x}\,u_- && \text{(2a)}\\ a^2 &= (39-x)\,u_+ + (7-x)\,u_- && \text{(2b)}\\ &= \tfrac{1}{2}[(39-x)+(7-x)] \\ &\qquad + \tfrac{1}{2}[(39-x)-(7-x)]\,u && \text{(2c)}\\ &= (23-x)+16u\,. && \text{(2d)} \end{aligned}$$

So, let's recap. We got this last form for a^2 by first squaring a to get a^2, and then converting that to standard form. Next, we'll convert a to standard form and then square that. After that, we'll equate, respectively, the complex and uniplex parts of both forms.

$$a = \sqrt{39-x}\,u_+ + \sqrt{7-x}\,u_- \tag{3a}$$
$$= \tfrac{1}{2}[\sqrt{39-x} + \sqrt{7-x}] \tag{3b}$$
$$+ \tfrac{1}{2}[\sqrt{39-x} - \sqrt{7-x}]\,u \tag{3c}$$
$$a = 4 + \tfrac{1}{2}B\,u\,. \tag{3d}$$

where we used (1) and where

$$B \equiv \sqrt{39-x} - \sqrt{7-x}\,. \tag{4}$$

Next, we square a:

$$a^2 = \left(16 + \tfrac{1}{4}B^2\right) + 4B\,u\,. \tag{5}$$

On equating the uniplex part of (2d) with the uniplex part of (6), we have that

$$16 = 4B\,, \tag{6}$$

hence,

$$B = 4\,. \tag{7}$$

Now, if we equate the complex part of (2d) with the complex part of (6), we have that

$$23 - x = 16 + \tfrac{1}{4}B^2\,. \tag{8}$$

On solving these last two equation together for x, we get

$$x = 3\,. \tag{9}$$

And this value of x does satisfy Eq. (1).