# Olympiad Problem 152

P. Reany

March 7, 2026

Every good mathematician is at least half a philosopher,
and every good philosopher is at least
half a mathematician.
— Gottlob Frege

```
Source:  https://www.youtube.com/watch?v=kDLP3lZh2TI
Title: ALGEBRA CHALLENGE
 Presenter: Maths Simplified Solutions
```

## 1 Problem

Given the relation

$$x^{332} + x^{-332} = 963\,, \tag{1}$$

find the real values of

$$\phi = x^{166} - x^{-166}\,. \tag{2}$$

## 2 Solution

Let's solve this problem by use of the unipodal algebra.

Let

$$\begin{aligned} a &= x^{166}u_+ + x^{-166}u_- && \text{(3a)}\\ &= \tfrac{1}{2}(x^{166} + x^{-166}) + \tfrac{1}{2}(x^{166} - x^{-166})\,u && \text{(3b)}\\ &= \tfrac{1}{2}(x^{166} + x^{-166}) + \tfrac{1}{2}\phi u\,, && \text{(3c)} \end{aligned}$$

where we use (2).

On squaring (3a), we get

$$a^2 = x^{332}u_+ + x^{-332}u_-\,. \tag{4}$$

Expressed in standard form, this become

$$a^2 = \tfrac{1}{2}(x^{332} + x^{-332}) + \tfrac{1}{2}(x^{332} + x^{-332})\,u \tag{5}$$

$$= \tfrac{1}{2}(963) + \tfrac{1}{2}(x^{332} + x^{-332})\,u\,. \tag{6}$$

Now, on squaring (3c), we have that

$$a^2 = \tfrac{1}{4}(x^{166} + x^{-166})^2 + \tfrac{1}{4}\phi^2 + \tfrac{1}{2}(x^{166} + x^{-166})\phi u\,. \tag{7}$$

But,

$$(x^{166} + x^{-166})^2 = (x^{332} + x^{-332} + 2) = 965\,, \tag{8}$$

where we used (1). So, on using this last result in (6), we get

$$a^2 = \tfrac{1}{4}(965) + \tfrac{1}{4}\phi^2 + \tfrac{1}{2}(x^{166} + x^{-166})\phi u\,. \tag{9}$$

On equating the scalar part of this last equation with the scalar part of (6), we get

$$\tfrac{1}{2}(963) = \tfrac{1}{4}(965) + \tfrac{1}{4}\phi^2\,, \tag{10}$$

which we can easily solve for $\phi$, getting

$$\phi = \pm 31\,. \tag{11}$$