Cosmology Notes for L. Susskind's Lecture Series (2013), Lecture 7

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Abstract

This paper contains my notes on Lecture Seven of Leonard Susskind's 2013 presentation on Cosmology for his Stanford Lecture Series. These notes are meant to aid the viewer in following Susskind's presentation, without having to take copious notes. The fault for any inaccuracies in these notes is strictly mine.

1 The temperature history of the universe

We are interested in discovering the temperature history of the universe. The notion of temperature only has a well-defined meaning when a substance in thermal equilibrium. At one stage of the early universe (prior to transparency), equilibrium will occur from the mutual scattering of electrons, protons, and photons. Thermal equilibrium will be maintained so long as the universe expands more slowly that the time scale at which the 'fluid' can remain in equilibrium.

Assume we have a box into which photons are admitted by laser light through a small hole. In this case, the photons will bounce endlessly off the walls along their input tracks, and will not come to equilibrium.

To come into equilibrium they have to scatter off each other. However, photons rarely scatter off each other. On the other hand, if we admit charged particles into the box, the photons will scatter rapidly. With a high electron density, the internal energy will come to equilibrium and then we can say that the interior of the box has a temperature.

Now, the universe is electrically neutral. So, in the mix of photons, electron, and protons, the number of electrons is equal to the number of protons, or

$$N_{e^-} = N_{p^+} \,. \tag{1}$$

For thermal equilibium to occur, the electrons and protons cannot be in the form of neutral hydrogen, for they will not scatter photons well in that state. In this case, the universe of today is not in thermal equilibrium, and thus we cannot assign it a temperature.

The distribution of frequencies is determined by the temperature according to the blackbody radiation curve. We will need the relation

$$\lambda\nu = c \,. \tag{2}$$

Energy intensity is given as

$$I(T,\nu)\, dV\, d\nu \,. \tag{3}$$

We can use dimensional analysis to calculate intensity, which is the energy per unit volume, per unit frequency.

$$I = \left[\frac{Et}{V}\right][T] \,. \tag{4}$$

So,

$$I = \frac{2Tk_B\nu^2}{c^2}\,. \tag{5}$$

But this formula doesn't work at high frequencies: the Ultraviolet Catastrophe.

To get the correct formula, we need to introduce a new universal constant, Planck's constant. Then the formula is

$$I = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/k_BT}-1}\,. \tag{6}$$

For small ν (large λ)

$$I = \frac{2h\nu^3}{c^2}\frac{k_BT}{h\nu} = \frac{2\nu^2 k_BT}{c^2}\,. \tag{7}$$

For large ν,

$$I \sim \frac{2h\nu^3}{c^3}e^{-h\nu/k_BT}\,, \quad \lambda < \frac{hc}{k_BT}\,. \tag{8}$$

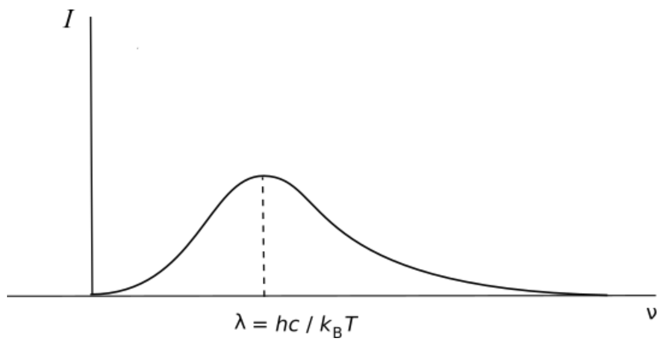

Figure 1. A thermal radiation curve.

The "thermal wavelength" is given by

$$\lambda = \frac{hc}{k_BT}\,. \tag{9}$$

Therefore, most of the energy is huddled around a wavelength that varies as $1/T$. Applying this to the universe, the background wavelength is about a millimeter, which corresponds to a temperature of about 3 K.

$$[I] = \left[\frac{E}{\nu V}\right] = \left[\frac{m\ell^2}{t^2}\left(\frac{1}{t}\right)^{-1}\frac{1}{\ell^3}\right]\,. \tag{10}$$

But

$$\begin{aligned}[I] &\sim \left[\frac{m}{\ell t}\right]\left[\frac{m\ell^2}{t^2}\right]\left[\frac{2Tk_B\nu^2}{c^3}\right]\\ &= \left[\frac{m\ell^2}{t^2}\left(\frac{1}{t^2}\right)^{-1}\frac{t^2}{\ell^2}\right]\\ &= \left[\frac{m}{t^2}\right]\,. \end{aligned} \tag{11}$$

Note: There is a missing factor of c somewhere!

Hence,

$$I \propto \nu^3 \frac{1}{e^{h\nu/k_b T} - 1} = \left(\frac{\nu}{T}\right)^3 \frac{1}{e^{h\nu/k_b T} - 1} T^3 \,. \tag{12}$$

The factors

$$\left(\frac{\nu}{T}\right)^3 \frac{1}{e^{h\nu/k_b T} - 1} \tag{13}$$

give the shape function F of $I(\nu/T)$:

$$F\left(\frac{\nu}{T}\right) = F\left(\frac{1}{T\lambda}\right) \,. \tag{14}$$

We can use this function to rescale the curve at various temperatures. As the universe expands the shape of the radiation curve remains the same.

When the temperature drops too low inside the box, the e^- and p^+ combine to form hydrogen, which is a poor scatterer, and therefore radiation lives independently of these atoms.

Assumptions:

1. Assume box is initially at high temperature.
2. Expand the box and the temperature drops.
3. Hydrogen forms, radiation decouples from the radiation.
4. Hence, no more thermal equilibrium.
5. But the radiation curve retains its same shape.

The univesal radiation for today has the blackbody form which is the legacy of the blackbody it had when matter and radiation decoupled.

Fact: Most of the photons in the universe are on the order of a millimeter wavelength and are from the CMB.

$$\frac{\lambda_{\substack{\text{today}\\\text{at peak}}}}{\lambda_{\text{decoupling}}} = \frac{a_{\text{today}}}{a_{\text{decoupling}}} = \frac{T_{\text{decoupling}}}{T_{\text{today}}} = \frac{4000\,\text{K}}{3\,\text{K}} \,. \tag{15}$$

From direct measurement, we know that the ratio of photons to electrons is

$$\frac{N_{\text{photons}}}{N_{\text{electrons}}} = 10^8 \,. \tag{16}$$

The probability that a photon has an energy value

$$e^{-\epsilon/k_b T} \,. \tag{17}$$

The ionization energy of the hydrogen atom is about 13 eV. Per atom the number of photons of ionizing strength is

$$\# = 10^8 e^{-\epsilon/k_b T} \,. \tag{18}$$

If this number is of order unity then ionization should occur.

Now,

$$10^8 \approx e^{20} \,. \tag{19}$$

Therefore,

$$\frac{\epsilon_{\text{ionization}}}{k_B T} \approx 20\,. \tag{20}$$

Thus

$$k_B T \approx \frac{\epsilon_{\text{ionization}}}{20}\,. \tag{21}$$

Hence, due to the large number of photons, ionization will occur well below the kinetic energy needed for ionization, because of the statistical spread of energies.

$$\frac{T_{\text{decoupling}}}{T_{\text{today}}} \approx 1000\,. \tag{22}$$

Thus

$$\frac{a_{\text{today}}}{a_{\text{decoupling}}} \approx 1000\,. \tag{23}$$

$$\rho_{\text{matter}} \to \rho_{\text{m}}/a^3\,. \tag{24}$$

while

$$\rho_{\text{rad}} \to \rho_{\text{r}}/a^4\,. \tag{25}$$

The crossover between a radiation-dominated and matter-dominated universe occurred with the energy of each photon being about 10^{-4} eV and the energy per proton being about a billion electron volts:

$$\frac{\rho_{\text{matter}}}{\rho_{\text{radiation}}} \approx \frac{10^{13}}{10^8 \text{ photons per proton}} = 10^5\,, \tag{26}$$

which is upgraded to 10^6 when we include dark matter.

Hence, if we go back in time to when

$$a(t) = 10^{-6} a(\text{today})\,, \tag{27}$$

(and the temperature is a million times larger) then the energy densities of matter and radiation were the same.

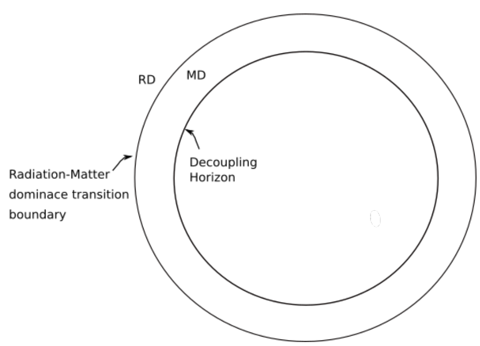

Figure 2. Major transtitional events that occurred before the decoupling horizon.

When the temperature gets to be about 10^{10} K

$$\frac{a_T}{a(t)} \approx 10^{40} \tag{28}$$

the temperature would be 10^{14} times larger, at which time the characteristic photon energy would be about a million electron volts: $\epsilon_{\text{photon}} \approx 0.5$ MeV, which is roughly the mass of a single electron. So, if two such photons should collide, they could transition to a pair production of an electron and a positron. At a certain temperature range, the electrons, positrons, and photons come to thermal equilibrium, such that

$$N_e = N_{e^+} = N_\gamma \,. \tag{29}$$

Now, a rough and ready calculation. The number of electron today we'll call N_e. Since the number of positrons is thought to be very small, then $N_e \approx N_e - N_{e^+}$. Now, $N_e + N_{e^+} \approx N_\gamma$. So,

$$\frac{N_e - N_{e^+}}{N_e + N_{e^+}} = 10^{-8} \,. \tag{30}$$

The interpretation of this is that at one time in the history of the universe, for every 10^8 electron-positron pairs, there was one extra positron.

What happens if we go back to a time when the temperature is a factor of a thousand times hotter? At that temperature, we can get quarks and antiquark, protons and antiprotons. At this point, we have a soup of photons, electron, positrons, quarks, and antiquarks.