Cosmology Notes for L. Susskind's Lecture Series (2013), Lecture 9

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Abstract

This paper contains my notes on Lecture Nine of Leonard Susskind's 2013 presentation on Cosmology for his Stanford Lecture Series. These notes are meant to aid the viewer in following Susskind's presentation, without having to take copious notes. The fault for any inaccuracies in these notes is strictly mine.

1 Review

The Potential Energy Density V is a scalar field we looked at in the last lecture.

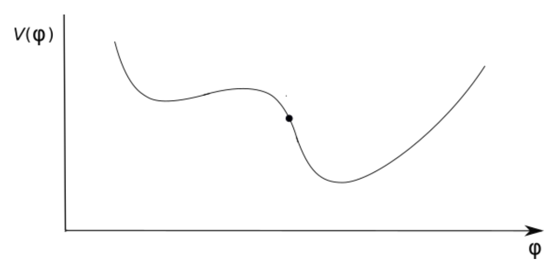

Figure 1. The Potential Energy Density V is a scalar field we looked at in the last lecture.

The total Energy Density requires both the kinetic and potential energies, taking the form of

$$E = \frac{\dot{\varphi}^2}{2} + V(\varphi)\,, \tag{1}$$

where we're using the form of energy density without the scale factor. The Lagrangian with this equation is

$$\mathscr{L} = \frac{\dot{\varphi}^2}{2} - V(\varphi)\,. \tag{2}$$

Applying the Euler-Lagrange equation to the energy density.

$$\frac{d}{dt}\frac{\partial \mathscr{L}}{\partial \dot{\varphi}} = \frac{\partial \mathscr{L}}{\partial \varphi} \tag{3}$$

we get

$$\ddot{\varphi} = -\frac{\partial V}{\partial \varphi} = F(\varphi)\,, \tag{4}$$

For our purposes in cosmology, we assume that because of the expansion of the universe, this potential function has been 'stretched out' and thus it must be a slowly varying function.

To get the energy in a box, we have to multiply the energy density by the volume of the box, which we take to be the cosmic scalar factor cubed.

$$E = a^3(t)[\,\frac{\dot{\varphi}^2}{2} + V(\varphi)\,]\,, \tag{5}$$

from which we can setup the Lagrangian:

$$\mathscr{L} = a^3(t)[\,\frac{\dot{\varphi}^2}{2} - V(\varphi)\,]\,. \tag{6}$$

This time the Euler-Lagrange equation gives us

$$\frac{d}{dt}\big(\dot{\varphi}a^3(t)\big) = -a^3(t)\frac{dV}{d\varphi}\,. \tag{7}$$

After a little computation, we have our 'equation of motion':

$$\ddot{\varphi} + 3H\dot{\varphi} = F\,, \tag{8}$$

where that factor $3H$ is referred to as the *viscosity coefficient*, and $3H\dot{\varphi}$ is referred to as the *cosmic friction term.*

On the condition that $\ddot{\varphi} = 0$, we have that

$$3H\dot{\varphi} = F\,, \tag{9}$$

and the object has reached its terminal speed,

$$\dot{\varphi} = \frac{F}{3H}\,. \tag{10}$$

We have the amazing result that we don't have energy conservation. The viscocity is sucking energy out of the system. This occurs from one point of view because the system is not time-invariant.

2 The Friedman Equation

The Friedman Equation again:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} = \frac{8\pi G}{3}\left[\dot{\varphi}^2/2 + V(\varphi)\right]. \tag{11}$$

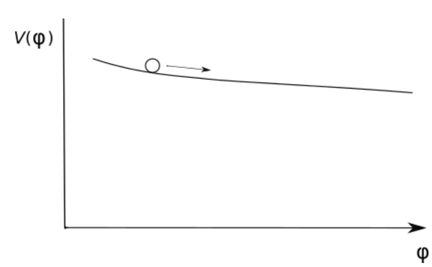

Figure 2. The Potential Energy Density V (the 'Inflaton field') is a slowly decreasing scalar field.

Next, we assume that V is a slowly varying function of time: We assume that the terminal speed is very slow, therefore,

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} V(\varphi)\,. \tag{12}$$

If we further treat V as a constant, then H is a constant. That gives us

$$\frac{da}{dt} = Ha\,. \tag{13}$$

So,

$$a = e^{Ht} \sim e^{60}\,. \tag{14}$$

Thus,

$$H = \left[\frac{8\pi G}{3} V(\varphi)\right]^{1/2}. \tag{15}$$

This equation would model the very early universe.

3 Alan Guth and Monopoles

In 1980, Alan Guth proposed this theory of stretching the universe:

1) To explain why magnetic monopoles had not been discovered in the universe (not including artificially made). FYI, to make them in the lab would not be easy because they are very heavy particles. However, the early universe is thought to have had a lot of energy hanging around to make magnetic monopoles in pairs, which are very stable. So, if they were made in the early universe, where are they? (In fact, why isn't the universe filled with ancient magnetic monopoles that would have used up all the initial kinetic energy?)

2) To explain why the universe at the time of decoupling of light from matter at about 380,000 years from the start, which required an expansion factor of at least e^{60}, which accounts for both the lack of monopoles (or the dilution of them) and the smoothness of the CMB.

Note: If primitive monopoles had survived into the current age and had found their way into galxaies, they would have discharged the magnetic fields of the galxies, which they have not.

However, to end the rapid expansion, the φ curve had to fall off rapidly at some point in time.

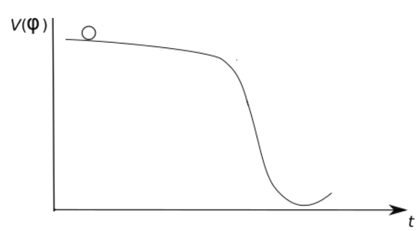

Figure 3. Eventually, $V(\varphi)$ curve had to fall off rapidly. We don't know why this curve has this shape. At the low point of the curve, the universe reheated, but at a temperature lower than that that could produce monopoles.

As we move along the curve, H gets smaller. $V(\varphi)$ was converted to the velocity of the field — and that caused the Big Bang energy expansion.

If inflation particles were created, their decay could be the source of the kinetic energy at the initial part of the Big Bang. One theory is that today's dark energy (cosmological constant) is the residual of $V(\varphi)$, which is 120 orders of magnitude smaller than the initial height of $V(\varphi)$.

Apparently the universe began 'hot' when monopoles could have been created. But, during the expansion, the universe cooled, though at the end of the expansion it re-heated, but at a temperature less than that required to generate more monopoles.

The $V(\varphi)$ shape is designed to fit the empirical data. At the current time, there is no deeper theory to explain its shape. Our best meta-explanation is to invoke *The Anthropic Principle*. At the current rate of expansion of the universe, it doubles in size every 10 billion years.

It's interesting that Susskind claims that in a trillion years, the CMB will virtually disappear and all galaxies except the Milky Way and Andromeda will recede past the cosmic horizon. The consequence of this will be that there will no longer be any visible evidence that the inhabitants of that time will know that the universe is in exponential inflation. (I wonder if, then, that we of today have a 'more preferred' view of the universe than will those people of that future date. In some sense, information will have been lost.)

At what distance from the earth are objects moving at the speed of light? $D = c/H$ is where the horizon is, which is about 15 billion light years.

4 Caustics

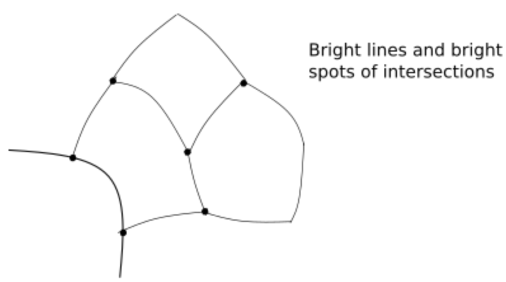

Figure 4. The curves represent strings of galaxies. The points are the bright spots of their intersections.

In a 1-d world, caustics would be localized high-intensies. In 2-d, they'd be lines of high-intensities.

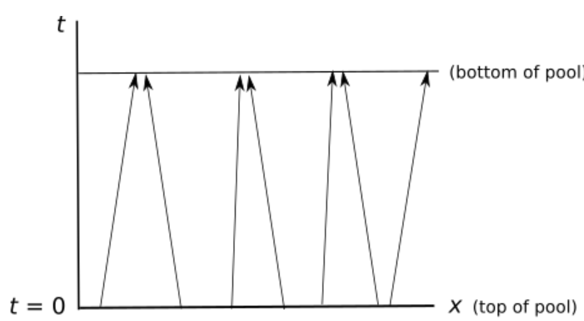

Figure 5. If the top of the pool is wavy, caustics will form at the bottom.

For the density of the particles on the x-axis, we have

$$\frac{dn}{dx} \equiv 1\,, \tag{16}$$

where we chose to set the ratio to unity to avoid dealing with unessential features.

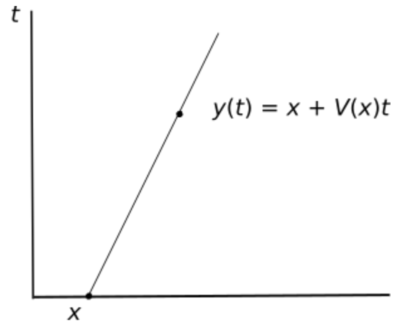

Figure 6. Choosing $y = y(t)$ is a relabeling.

$$\frac{dy}{dx} = 1 + V'(x)t\,. \tag{17}$$

From this we have that

$$\frac{dn}{dy} = \frac{dn/dx}{dy/dx} = \frac{1}{1 + V'(x)t}\,. \tag{18}$$

Bright spots occur when

$$1 + V'(x)t \equiv 0\,. \tag{19}$$

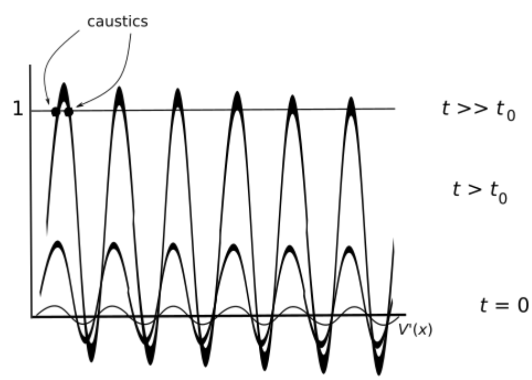

Figure 7. This is a plot of $-V'(x)$. I drew the curves to look periodic but they really aren't. In 2-d, reference Fig. 4.

So far, we haven't mentioned the role of gravity in the expansion. After the expansion and its smoothing effect, where did the initial non-smoothness come from?