A Few Problems from *Analytical Mechanics*, Chapter 7

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1 Introduction

Here we present the solutions to a few problems from the physics textbook *Analytical Mechanics*, Chapter 7, Grant R. Fowles (Holt, Rinehart, and Winston), New York, 1962, p. 152–3.

Problem 1. A gun of mass M fires a bullet of mass m. Find the velocity of the bullet and the recoil velocity of the gun in terms of the energy E of the explosive charge.

Solution 1. To solve this simply, I will make the assumption that all the explosive energy E went into the kinetic energy of the gun and bullet. Let the mass of the bullet be m and the mass of the gun be M. Also, the speed of the bullet be v and the speed of the gun be V. Then we have that

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}MV^2\,. \tag{1}$$

Now, we also need a relationship between the two speeds which comes from the conservation of momentum, which is zero, as there are no external forces acting:

$$m\mathbf{v} + M\mathbf{V} = 0\,. \tag{2}$$

From this we get the relation between the speeds:

$$V = \frac{mv}{M}\,. \tag{3}$$

On squaring this, we get

$$V^2 = \frac{m^2}{M^2}v^2 = \frac{2m}{M^2}\big(\tfrac{1}{2}mv^2\big)\,. \tag{4}$$

Substituting into this from (1), we get

$$V^2 = \frac{2m}{M^2}\big(E - \tfrac{1}{2}MV^2\big)\,. \tag{5}$$

Solving this for V, we get

$$V = \left[\frac{2mE}{M(m+M)} \right]^{1/2} . \tag{6}$$

By a similar calculation, we get for v:

$$v = \left[\frac{2ME}{m(m+M)} \right]^{1/2} . \tag{7}$$

Problem 3. A bullet of mass m and speed v_0 is fired into a block of wood of mass M resting on a rough horizontal table If μ is the coefficient of sliding friction between the block and the table, how far will the block slide before coming to rest? (We'll call the distance the block slides D.)

Solution 3. This problem is classified as completely inelastic. Thus, macroscopic kinetic energy is not conserved, but linear momentum is. Just regarding magnitudes, we have

$$mv_0 = (M+m)V \, . \tag{8}$$

Now, we make the approximation that he block will not move any appreciable amount while receiving the bullet, after which the Block = block+bullet will slide along the table top with a frictional force f_r acting in the direction opposite to the velocity of the block. And we have that

$$f_r = -\mu N \, , \tag{9}$$

where N is the normal force on the Block by the table top. For this problem, the normal force is equal and opposite to the weight of the Block: $N = (M+m)g$. Hence, the magnitude of the acceleration a of the frictional force on the Block is

$$a = \frac{f_r}{M+m} = -\mu g \, . \tag{10}$$

But we know from prior work in mechanics that

$$V_f^2 = v_i^2 + 2aD \, , \tag{11}$$

where V_f is the final speed, which is zero, and $v_i = V = v_0 m/(M+m)$, hence

$$D = -\frac{v_i^2}{2a} = \frac{v_i^2}{2\mu g} = \frac{(v_0 m/(M+m))^2}{2\mu g} = \frac{v_0^2}{2\mu g} \Big(\frac{m}{M+m} \Big)^2 . \tag{12}$$

Problem 12. A uniform heavy chain of length a hangs initially with part of length b hanging over the edge of the table. The remaining part, of length $a-b$, is coiled up at the edge of the table. If the chain is released, show that the speed of the chain when the last link leaves the end (edge) of the table is $[\,2g(a^3 - b^3)/3a^2\,]^{1/2}$.

Solution 12. First, a comment. I didn't get this answer, and neither did anyone else who published an answer to this problem on the Internet (that I found), such as on https://physics.stackexchange.com.

The first of my assumptions about this problem is that there is no friction between the tabletop and the chain. The second assumption is that the dynamics of the links in the vacinity of the table edge are to be ignored. The next assumption is that the linear mass density of the chain ρ is constant over the length of the chain. (Of course this is an absurd approximation at face value. What we really seem to be modeling is a thin strip of highly flexible material. But, we'll go with this approximation.) The last assumption is that every part of the chain has the same speed and acceleration (in magnitude) as every other part of the chain.

So, I'll break up this chain into two logical parts: The part on the tabletop, and the hanging part. The entire chain is accelerated by the sum of all the unbalanced external forces on it. The forces on the part of the chain on the tabletop are its weight (of the part on the tabletop) and the normal force of the table on that part of the chain, and these two forces are assumed to cancel each other out. Thus, at any instant, the total unblanced external force on the chain is equal to the force of gravity experienced by only the hanging part of the chain, and that is equal to the mass of the hanging part times the acceleration of gravity g. Thus,

$$M\ddot{x} = \sum_{\substack{\text{Hanging part}\\ \text{of the chain}}} \text{Forces}\,. \tag{13}$$

But, $M = \rho a$. And, since only gravity acts on the overhanging part of the chain, whose length is $y(t)$, where I'm taking the $+y$ direction as down. Therefore, the last equation becomes

$$a\rho\ddot{x} = \rho g y(t)\,. \tag{14}$$

Oh really? Well, we also have that $\ddot{x} = \ddot{y}$. On making this substitution and dividng through by $a\rho$, we get

$$\ddot{y} = \frac{g}{a}y\,. \tag{15}$$

I'm going to try to solve this equation by the path less taken, by multiplying through by $\dot{y}$, yielding

$$\dot{y}\ddot{y} = \frac{g}{a}y\dot{y}\,. \tag{16}$$

Then on reconfiguring, we get

$$\tfrac{1}{2}D_t(\dot{y}^2) = \frac{g}{a}\tfrac{1}{2}D_t(y^2)\,. \tag{17}$$

Now, we're going to integrate this equation on both sides by first denoting the time for the last link to clear the tabletop as T. Then, at time $t = 0$, $y(0) = b$,

$\dot{y}(0) = 0$, and at time $t = T$, $y(T) = a$, $\dot{y}(T) = v$, the value we're looking for. Then, on integrating, we get

$$\dot{y}^2\Big|_{t=0}^{T} = \frac{g}{a} y^2\Big|_{b}^{a}, \tag{18}$$

or

$$v^2 = \frac{g}{a}(a^2 - b^2)\,. \tag{19}$$

Therefore,

$$v = [\,\frac{g}{a}(a^2 - b^2)\,]^{1/2}\,. \tag{20}$$

To get the book answer, one needs, instead of (16), to have

$$\dot{y}\ddot{y} = \frac{g}{a}\frac{y}{a} y\dot{y}\,, \tag{21}$$

though I don't know how to 'derive' this. Anyway, it reconfigures to:

$$\tfrac{1}{2} D_t(\dot{y}^2) = \frac{g}{3a^2} D_t(y^3)\,, \tag{22}$$

which then becomes

$$v^2 = \frac{g}{3a^2}(a^3 - b^3)\,, \tag{23}$$

and so on.

Anyway, I got (19) instead of (23).