

Deriving the Ideal Gas Law

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June 10, 2021

Abstract

In this short note, I wish to produce a heuristic derivation of the Ideal Gas Law from Boyle's Law, Charles's Law, Avogadro's Law, and Gay-Lussac's Law.

1 Proof

My heuristic derivation of the **Ideal Gas Law**

$$PV = nRT, \tag{1}$$

will follow from the ABC's of early chemistry, which, presented in alphabetical order are: Avogadro's Law, Boyle's Law, Charles's Law, and Gay-Lussac's Law. (Remember that the above equation is concerned with properties of a contained gas.) The symbols in this equation are P for pressure, V for volume, T for temperature, n for the number of moles in the gas, and R is known as the **universal gas constant**, whose numeric value is not important to this derivation.

The proof here will require the algebraic manipulation of these four gas laws, starting with **Avogadro's Law**:

$$\frac{V}{n} = c_1, \tag{2}$$

where c_1 is a constant and variables P and T are held constant.

Next is **Boyle's Law**:

$$PV = c_2, \tag{3}$$

where c_2 is a constant and variables n and T are held constant.

Next is **Charles's Law**:

$$\frac{V}{T} = c_3, \tag{4}$$

where c_3 is a constant and variables P and n are held constant.

Last is **Gay-Lussac's Law**:

$$\frac{P}{T} = c_4, \tag{5}$$

where c_4 is a constant and variables V and n are held constant.

At this point, the algebraic derivation of (1) is sort of a puzzle, where the constant R is a function of the constants c_1 , c_2 , c_3 , and c_4 , or their inverses.

Now, my intuition has been that there should exist a straight-forward algebraic proof of the Ideal Gas Law from just Boyle's Law, Charles's Law, and Avogadro's Law, but as of this moment, I haven't found it. However, there is a simple proof of it from Avogadro's Law and Gay-Lussac's Law. Just multiply them together to get

$$\frac{PV}{nT} = c_1 c_4. \quad (6)$$

Now set $c_1 c_4 = R$ and then multiply through by nT and the result is (1).

To obtain a proof that doesn't require Gay-Lussac's Law, just prove Gay-Lussac's Law in terms of Boyle's Law and Charles's Law, if you're able to find one.

From a YouTube video (Kisembo Academy), I found this approximate version of an algebraic proof: Multiply Eqs. (3), (4), and (5) together to get

$$\frac{P^2 V^2}{T^2} = c_2 c_3 c_4. \quad (7)$$

Then, we set $c_0^2 \equiv c_2 c_3 c_4$ and use this in the above equation, after taking the square root of both sides, we get

$$PV = c_0 T. \quad (8)$$

What now? We can set $c_0 \equiv cR$, and write

$$PV = cRT. \quad (9)$$

That's closer, but it doesn't explicitly contain the variable n .

2 Conclusion

So, just how convincing is the algebraic derivation of the Ideal Gas Law presented here? I'd call it convincing, but I'd prefer to provide a proof that doesn't use Gay-Lussac's Law.

Alternatively, there is a convincing proof of the Ideal Gas Law employing kinetic theory, which is based on the principles of Newtonian mechanics applied to a contained ideal gas in equilibrium.