The Wheatstone Bridge

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Abstract

This paper presents the Wheatstone Bridge in its 'simplest' form.

The purpose of the Wheatstone Bridge is to measure the resistance of an unknown resistance, which I have labeled as R_x in Fig. 1. We need a formula, and to get that formula, all we will use is Ohm's Law. Other derivations use Kirchoff's Law.[1] The resistances of resistors R_1 and R_2 are selected before starting the measurement of R_x.

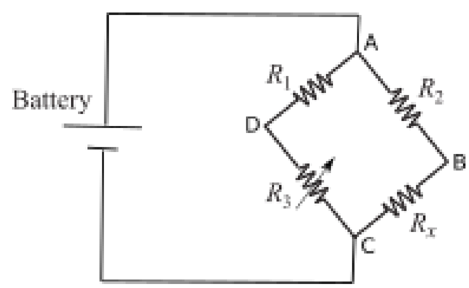

Figure 1. The so-called Wheatstone Bridge. The resistor labeled R_3 is a variable resistor, called a *rheostat*. R_x is some object of unknown resistance that we want to measure by this setup.

The Wheatstone Bridge is both clever and simple. We need only very little theory to understand it, that being Ohm's Law. Let P and Q be any two distinct points in an electronic circuit. Then the voltage drop between these two points is related to the resistance R and current I between these two points as

$$V = IR\,. \tag{1}$$

For a Wheatstone Bridge to work, we bridge the gap between points D and B with a galvanometer/ammeter and the adjust the rheostat R_3 until there is no current flowing through it. At this point, certain relationships must be true.

[1] I suppose that one could argue that I use Kirchoff's Law to prove that the current through AD is equal to the current through DC, etc.

First, the currect flowing from A to D must be the same as the current flowing from D to C. In other words,

$$I_1 = I_3\,. \tag{2}$$

By similar reasoning,

$$I_2 = I_x\,. \tag{3}$$

Second, since there is no current flowing between points D and B, even though they are connect by a conductor, then the potential at D must equal the potential at B. But that implies that the potential difference between points A and D must equal the potential difference between point A and B, or $V_{\text{AD}} = V_{\text{AB}}$. Thus, by Ohm's Law

$$I_1R_1 = I_2R_2\,. \tag{4}$$

And by similar reasoning,

$$I_3R_3 = I_xR_x\,. \tag{5}$$

On dividing (5) by (4), we get

$$\frac{I_3R_3}{I_1R_1} = \frac{I_xR_x}{I_2R_2}\,. \tag{6}$$

We can now simplify by using (2) and (3):

$$\frac{R_3}{R_1} = \frac{R_x}{R_2}\,. \tag{7}$$

Solving for R_x, we get the formula we need.

$$R_x = \frac{R_2R_3}{R_1}\,. \tag{8}$$