Galilean Transformation of Coordinates

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Abstract

The simplest version of the Galilean Transformation of Coordinates is proven here, by beginning with the heuristic of *Scheme*.

1 Setup

In Fig. 1, we see the typical representation of the Galilean Transformation of Coordinates, whose equations are

$$x' = x - vt \tag{1a}$$
$$y' = y \tag{1b}$$
$$z' = z \tag{1c}$$
$$t' = t\,. \tag{1d}$$

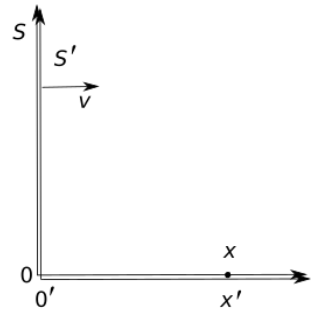

Figure 1. Depicted are two reference frames, S and S', that have their three corresponding axes aligned. We regard frame S as fixed and S' as moving to the right at speed v. Equations $y' = y$ and $z' = z$ hold because of symmetry. Eq. $t' = t$ is due to the common-sense notion of absolute time shared between ref. frames. These frames are coincident at times $t' = t = 0$, at which $x' = x$.

At some finite time t in frame S, frame S' has move to the right a distance vt. In other words, the origin $0'$ has moved along S's x-axis a distance vt away from origin 0.

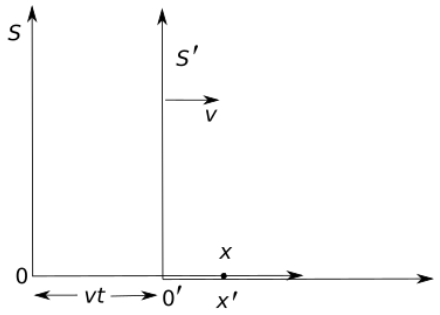

Figure 2. At some finite time t in frame S, frame S' has move to the right a distance vt. That is, the origin $0'$ has moved a distance vt away from origin 0 in S.

2 Solution to the Problem in Scheme

Scheme is a set of heuristics designed to aid in problem solving, especially in college algebra. One of its most basic rules is

> Every total is equal to the sums of its parts.

To use this heuristic is easy enough: We simply look for either a total or some parts. Having found either one, it's then obvious what to do next.

Regarding Fig. 2, it's clear that the total distance of point labeled x in frame S is x and that it can be divided into two parts: the distance from 0 to $0'$ (which is vt) and the distance of $0'$ to x' (which is x'). From this we get the equation

$$x = vt + x' \,. \tag{2}$$

Using simple algebra, we get

$$x' = x - vt \,, \tag{3}$$

which is what we needed to show.