

Special Relativity Problem 1

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Abstract

Here we derive the scalar energy-momentum equation with some algebra.

1 Statement of the Problem

We are asked find the relativistic scalar equation for the energy of a mass particle of mass m :

$$E^2 = \mathbf{p}^2 c^2 + m^2 c^4, \quad (1)$$

starting with relativistic energy

$$E = \gamma m c^2, \quad (2)$$

and relativistic momentum

$$\mathbf{p} = \gamma m \mathbf{v}, \quad (3)$$

where

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}. \quad (4)$$

2 Solution to the Problem

I prefer to work with the new variable

$$\beta = \frac{v}{c}, \quad (5)$$

so,

$$\gamma^2 = \frac{1}{1 - \beta^2}. \quad (6)$$

Now, on squaring (4), we get

$$\mathbf{p}^2 = \gamma^2 m^2 \mathbf{v}^2. \quad (7)$$

On multiplying this last equation through by c^2 , we get

$$\mathbf{p}^2 c^2 = \frac{1}{1 - \beta^2} m^2 c^4 \beta^2. \quad (8)$$

On replacing the second β^2 in the last equation by the equivalent expression

$$\beta^2 = -[(1 - \beta^2) - 1], \quad (9)$$

we have

$$\begin{aligned} \mathbf{p}^2 c^2 &= -m^2 c^4 + \frac{1}{1 - \beta^2} m^2 c^4 \\ &= -m^2 c^4 + \gamma^2 m^2 c^4. \end{aligned} \quad (10)$$

On rearranging this equation and using (2), we get (1).