de Sitter's Proof of the Constancy of the Speed of Light

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Abstract

In 1913, W. de Sitter's published a proof of the constancy of the speed of light using light emitted from a binary star system. De Sitter investigated the consequences if the speed of light were dependent on the speed of the source relative to the observer. The proof given here will be similar to de Sitter's, except that we will begin from the viewpoint of *Scheme*.

1 Setup

W. de Sitter[1] published the article "An Astronomical Proof for the Constancy of the Speed of Light," which produced a result that contradicts the observational result that a distant star's light, which is in orbit around a more massive object, appears only at a single point on its orbit around the central star at a time.

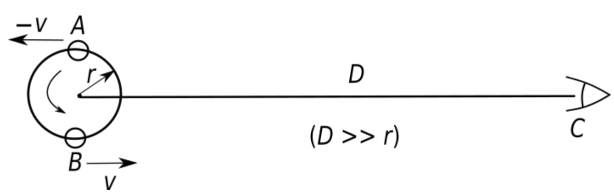

Figure 1. The big circle on the left represents the orbit of the lighter star around the heavier star of a binary pair. We assume that the speed of light relative to point C is the constant c when the source is at rest. We further assume that when the source moves away from the observer that we subtract off the speed of the source from the speed of light. Similarly, when the source moves towards the observer that we add on to c the speed of the source.

The word 'proof' is a bit strong. What this argument provides is a plausibility argument. If we assume that the speed of light propagation is affected by the

[1] "An Astronomical Proof for the Constancy of the Speed of Light," Physik. Zeitschr. 14, 429, (1913).

speed of its source relative to the frame in which it is being measured, then we get a plausible condition on which it is possible to 'see' an orbiting star appear at opposites points on its orbit at the same time, contrary to observation. Hence, we conclude that one or more of our assumptions is wrong, and that would be that the speed of light emitted by a source is dependent on the relative speed of that source compared to the frame measuring it.

At this point we make a bunch of 'reasonable' assumptions about the situation. First, that the binary system is far away from the observer on earth; second, that the radius r of the orbit (circular for simplicity) is small compared to the distance of the binary center D from earth.

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From observation of binary systems, we never see the orbiting star at two side of the orbit at the same time. What we will show is that if the speed of light is not a constant, independent of the speed of the source, then it is conceivable that the orbiting star could appear to be at both points A and B at the same time as seen at observer's point C.

2 Solution to the Problem

So, from the heuristics we have adopted in Scheme, can we find any totals to set equal to the sums of its parts? Yes, we can. To begin with, both light rays exiting the binary system must travel the same distance D (approximately) to get to the observer at C. We will need the simple kinematic equation that the time of transit for a particle in constant motion is the distance it travels divided bi its speed.

Now, let's make some convenient definitions for time intervals.

Let T_1 be the time it takes for light to go from point A to point C in the observer's frame.

Let T_2 be the time it takes for light to go from point B to point C in the observer's frame.

Let τ be the (delay) time it takes for the orbiting star to go from A to point B in the observer's frame.

Now, for the possibility that the observer at C can see light from both A and B at the same time, then the light has to be admitted from both A and B in such a way that they arrive at C at the same time. To accomplish this, we conclude that the time of flight of the light emitted at A, must equal the sum of the time it take light to travel from point B to point C plus the delay time it takes the star to travel in its orbit from A to B. As an equation, that gives us

$$T_1 = T_2 + \tau\,. \tag{1}$$

So, what is the travel time T_1?

$$T_1 = \frac{d_1}{v_1} = \frac{D}{c - v} \,. \tag{2}$$

Similarly, what is the travel time T_2 is

$$T_2 = \frac{d_2}{v_2} = \frac{D}{c + v} \,. \tag{3}$$

Solving for τ in (1) and using the last two equations we have that

$$\tau = \frac{D}{c - v} - \frac{D}{c + v} = 2D \frac{v/c^2}{1 - v^2/c^2} > 0 \,. \tag{4}$$

What this is saying is that there is a possible delay time τ that will allow the orbiting star to be seen at opposite sides of its orbit at the same time. This situation would be impossible if we assume that the speed of light is independent of the speed of its source.