Special Relativity Notes for L. Susskind's Lecture Series (2012), Lecture 1

P. Reany

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Abstract

This paper contains my notes on Lecture One of Leonard Susskind's 2012 presentation on Special Relativity for his Stanford Lecture Series. These notes are meant to aid the viewer in following Susskind's presentation, without having to take copious notes. The fault for any inaccuracies in these notes is strictly mine.

1 Getting Started

As a note, I introduce the use of the symbols S and S' to represent, respectively, the unprimed and primed inertial frames of reference. We begin with the *Galilean transformation* equations:

$$x' = x - vt\,, \tag{1a}$$
$$y' = y\,, \tag{1b}$$
$$z' = z\,, \tag{1c}$$
$$t' = t\,. \tag{1d}$$

The unprimed coordinates belong to the S frame and the primed coordinates belong to the S' frame.

The Principle of Relativity: The laws of physics are the same in all inertial reference frames.

Definition: An *inertial reference frame* is a) a frame in which the laws of Newton hold, or b) a frame in which free particles (no unbalanced external forces on them) travel in straight lines at constant speeds.

The Light Principle:[1] Every inertial reference frame will measure the speed of light to be the same value, designated as c.

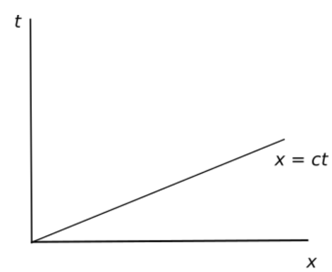

Figure 1. The line $x = ct$ is the path of a lightray.

[1] This is the name given to this principle by none other then Einstein himself Also see the Appendix.

Graphically, every line intersecting the t axis at a 'right' angle is a set of simultaneous points.

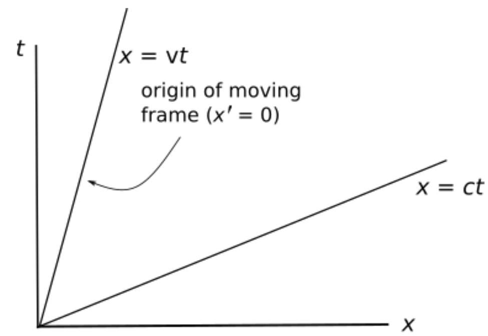

Figure 2. The position of the origin of S' as seen in S is given as $x = vt$.

Classically, the lightray as seen in S' entails that

$$x' = x_{LR} - vt = ct - vt = (c - v)t = (c - v)t' \,. \tag{2}$$

But this breaks the Light Principle.

By the way, a light ray moving in the opposite direction has equation

$$x' = -(c + v)t' \,. \tag{3}$$

Now we need to construct a line of simultaneity in the S' system. By construction we will set observers Fred, Mary, and Seymore on the x axis of S so that Mary is at the midpoint of the separation of Fred and Seymore, as in Fig. 3.

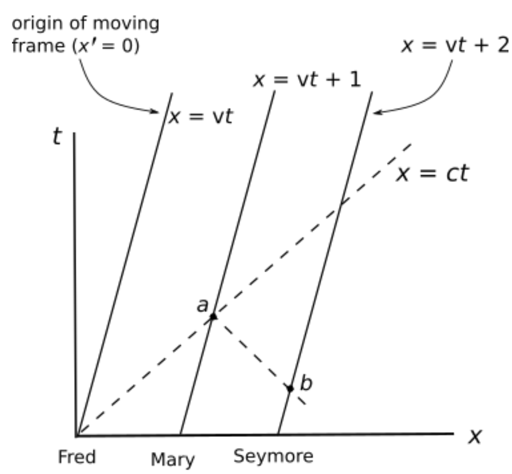

Figure 3. The lightrays are now dashed. Since Mary is equidistant between Fred and Seymore in S, she must also be in S'. Thus, to construct a point on Seymore's worldline simultaneous with the origin, we set a lightray from point a to point b on Seymore's worldline.

Before we continue, it's time to set $c = 1$ for good measure.

So, what do we know about the point a? First, since it's on the lightray, it conforms to $x = t$. Second, because it's on Mary's timeline, it conforms to $x = vt + 1$. Putting these together, we get

$$t = \frac{1}{1-v}\,, \tag{4a}$$

$$x = \frac{1}{1-v}\,. \tag{4b}$$

Now, the lightlike ray between a and b, has the familiar form (from analytic geometry)

$$x + t = \text{const}\,. \tag{5}$$

Employing (4a) and (4b), we can determine this constant:

$$x + t = \frac{2}{1-v}\,. \tag{6}$$

Solving for x_b and t_b, given that point b is also on Seymore's worldline, we get

$$t_b = \frac{2v}{1-v^2}\,, \tag{7a}$$

$$x_b = \frac{2}{1-v^2}\,. \tag{7b}$$

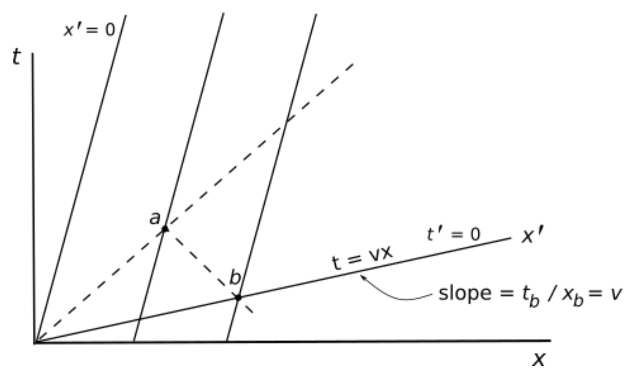

Figure 4. In the primed system we calculate the line of events simultaneous with the origin.

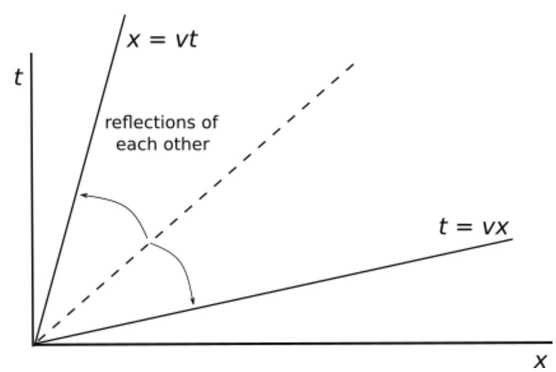

Figure 5. The line of simultaneity through the origin in the S' frame is the reflection of the moving frame's timeline through the lightline.

So, let's now see how to generalize the Galilean transformation equations, ignoring the y and z directions, as we'll assume they aren't changed. We try the ansatz:

$$x' = (x - vt)f(v)\,, \tag{8a}$$
$$t' = (t - vx)g(v)\,. \tag{8b}$$

Let's begin this by dealing with the scale factors $f(v)$ and $g(v)$. Here are some relevant pieces of information:

- At $x' = 0$, $x = vt$.
- At $t' = 0$, $t = vx$.

Next, we apply the Light Principle to both frames: The light path going through the origin is (with $c = 1$) $x = t$ and $x' = t'$ in the S and S' frames, respectively. Thus, using (8a) and (8b):

$$(x - vt)f(v) = (t - vx)g(v)\,. \tag{9}$$

And with $x = t$, we get

$$f(v) = g(v)\,. \tag{10}$$

This simplifies our transformation equations to

$$x' = (x - vt)f(v)\,, \tag{11a}$$
$$t' = (t - vx)f(v)\,. \tag{11b}$$

In addition, we can exchange the roles of the prime and unprimed variables, by replacing v with $-v$:

$$x = (x' + vt')f(v)\,, \tag{12a}$$
$$t = (t' + vx')f(v)\,. \tag{12b}$$

Now we can determine $f(v)$ by enforcing compatibility of the last two pairs of equations:

$$x = [(x - vt)f(v) + v(t - vx)f(v)]f(v)\,, \tag{13a}$$
$$t = [(t - vx)f(v) + v(x - vt)f(v)]f(v)\,. \tag{13b}$$

After a bit of cancellations, we get

$$x = [\,(1 - v^2)x\,]f(v)^2\,, \tag{14a}$$
$$t = [\,(1 - v^2)t\,]f(v)^2\,. \tag{14b}$$

From this, we conclude that $f(v) = 1/\sqrt{1 - v^2}$. I'm going to use a common notation

$$\gamma(v) = \gamma_v = \frac{1}{\sqrt{1 - v^2/c^2}}\,. \tag{15}$$

We can now finalize our (Lorentz) transformation equations to

$$x' = \gamma(x - vt)\,, \tag{16a}$$
$$t' = \gamma(t - vx/c^2)\,. \tag{16b}$$

It's fairly straightforward to see that γ is always greater than or equal to unity. The smaller the ratio v/c, the closer γ is to unity, and thus the closer the Lorentz transformation equations look like the Galilean transformation equations.

Now, let's invert the transformation equations:

$$x = \gamma(x' + vt')\,, \tag{17a}$$
$$t = \gamma(t' + vx'/c^2)\,. \tag{17b}$$

2 Special cases

Case 1. How does a meter stick at rest along the x axis in S measure out in the S' frame?

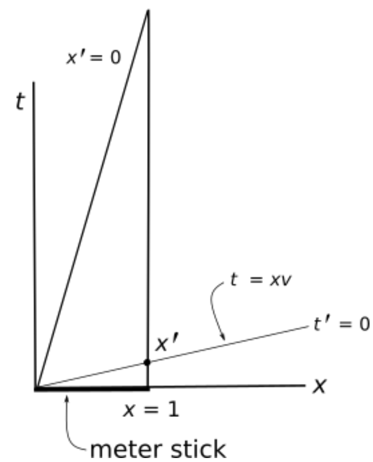

Figure 6. A meter stick lies on the x axis in S. What will S' measure its length x' to be?

To solve this problem, we notice that the point x' lies on two lines: One is the line $x = 1$ and the other is $t' = 0$. So, let's use our equations. So, applying these to our Lorentz transformation equations, we get ($c = 1$)

$$x' = \gamma(1 - vt)\,, \tag{18a}$$

$$0 = \gamma(t - v)\,. \tag{18b}$$

From this last equation we get that $t = v$, which can be substituted into the first equation to get

$$x' = \gamma(1 - v^2) = \sqrt{1 - v^2}\,. \tag{19}$$

Thus, according to observers in S', the moving meter stick is shorter by a factor of $\sqrt{1 - v^2}$.

Case 2. Now, let's reverse the situation. This time, the meter stick is at rest in S' along the x' axis. What, then, will its length x be recorded as in S?

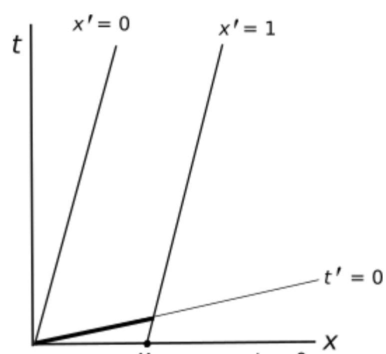

Figure 7. A meter stick lies at rest on the x' axis in S'. What will S measure its length x to be?

To solve this problem, similarly to our last derivation, we notice that the point x lies on two lines: One is the line $x' = 1$ and the other is $t = 0$. So, let's use our equations. So, applying these to our Lorentz transformation equations (17a) and (17b), we get ($c = 1$)

$$x = \gamma(1 + vt')\,, \tag{20a}$$
$$0 = \gamma(t' + v)\,. \tag{20b}$$

From this last equation we get that $t' = -v$, which can be substituted into the first equation to get

$$x = \gamma(1 - v^2) = \sqrt{1 - v^2}\,. \tag{21}$$

Thus, according to observers in S, the moving meter stick is shorter by a factor of $\sqrt{1 - v^2}$. And thus we see the perfect symmetry between the two frames of inertial reference.

Case 3.

How does a unit of time in the S' transform?

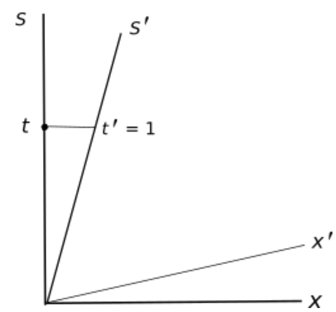

Figure 8. How will a unit of time in S' be measured in S?

We'll just use algebra this time. On the line $x' = x - vt = 0$, we have $t' = 1$. Thus, from (17b) and (17a)

$$vt = \gamma(0 + v)\,, \tag{22a}$$
$$t = \gamma(1 + 0)\,. \tag{22b}$$

Both of these equations tell us that $t = \gamma > 0$. Thus, S claims that the clocks in S' run slow, since in this case $t' < t$.

3 The Spacetime Interval

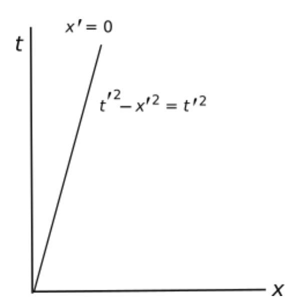

Figure 9. The spacetime interval.

From Fig. 9, we see how to define the proper time as the time between two events traversed (in principle) by a single clock. The spacetime interval (or its square) is an invariant number

$$x'^2 - t'^2 = x^2 - t^2 \,. \tag{23}$$

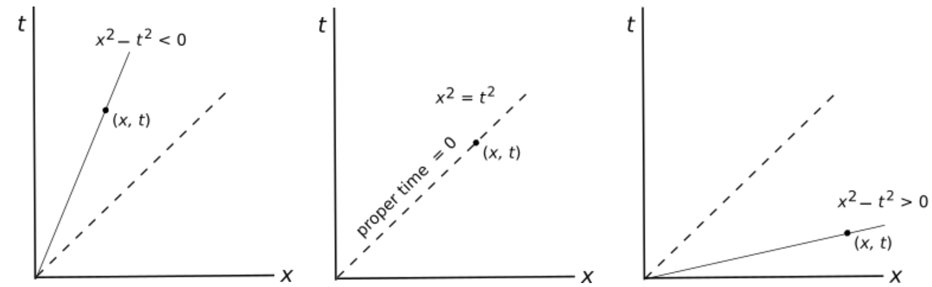

Figure 10. The spacetime interval in the left figure is timelike, in the middle figure is lightlike, and in the right figure is spacelike.

4 Appendix: The Light Principle

The special theory of relativity is an adaptation of physical principles to Maxwell-Lorentz electrodynamics. From earlier physics it takes the assumption that Euclidean geometry is valid for the laws governing the position of rigid bodies, the inertial frame, and the law of inertia. The postulate of equivalence of inertial frames for the formulation of the laws of Nature is assumed to be valid for the whole of physics (special relativity principle). From Maxwell-Lorentz electrodynamics it takes the postulate of invariance of the velocity of light in a vacuum (light principle).

Found in: "Fundamental ideas and problems of the theory of relativity," in Gerald Tauber's *Albert Einstein's Theory of General Relativity*, Crown Publ., 1979, p. 53.