Special Relativity Notes for L. Susskind's Lecture Series (2012), Lecture 2

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Abstract

This paper contains my notes on Lecture Two of Leonard Susskind's 2012 presentation on Special Relativity for his Stanford Lecture Series. These notes are meant to aid the viewer in following Susskind's presentation, without having to take copious notes. The fault for any inaccuracies in these notes is strictly mine.

1 Relativistic Addition of Velocities

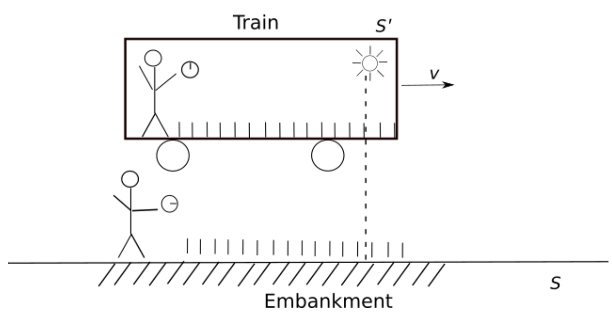

Figure 1. Train (frame S') is moving past the observer (frame S) standing on the ground. Both frames have meter sticks and clocks at rest in their own frames.

$$x' = \gamma_v(x - vt)\,, \tag{1a}$$
$$t' = \gamma_v(t - vx)\,, \tag{1b}$$

where

$$\gamma_v = \frac{1}{\sqrt{1-v^2}}\,. \tag{2}$$

We can also invert these equations:

$$x = \gamma_v(x' + -vt)\,, \tag{3a}$$
$$t = \gamma_v(t + vx)\,, \tag{3b}$$

The space coordinate of the primed origin in S' is, of course, $x' = 0$, which implies that

$$x - vt\,. \tag{4}$$

And his implies that the S' frame moves at speed v relative to S.

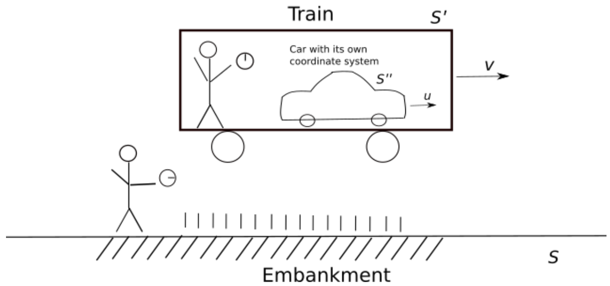

Figure 2. The little car (frame S'') moves in the same direction as the train at speed u relative to the floor of the train.

It's reasonable to ask what the observer on the embankment would claim that the speed of the car inside the train woudl be as measured relative to the embankment. Let's use the Lorentz transforamtion eqaution s to predict what the answer will be. So, to do that, let's write down the transformation equations between the S' and S'' frame:

$$x'' = \gamma_u(x' - ut')\,, \tag{5a}$$

$$t'' = \gamma_u(t' - ux')\,, \tag{5b}$$

where

$$\gamma_u = \frac{1}{\sqrt{1-u^2}}\,. \tag{6}$$

Next, we substitute out x' and t':

$$\begin{aligned} x'' &= \gamma_u[\gamma_v(x - vt) - u\gamma_v(t - vx)\,] \\ &= \gamma_u\gamma_v[\,(x - vt) - u(t - vx)\,]] \\ &= \gamma_u\gamma_v[\,x(1+uv) - t(v+u)\,] \end{aligned} \tag{7}$$

Obtaining the speed of the car relative to the embankment is mathematically equivalent to setting $x''(x,t) = 0$. When we do this in the last equation, we get the relation

$$x(1+uv) - t(v+u) = 0\,. \tag{8}$$

On dividing through by t, we get

$$w(1+uv) - (v+u) = 0\,, \tag{9}$$

where $w = x/t$. Finally, we get the interesting relationship:

$$w = \frac{v+u}{(1+uv)}\,. \tag{10}$$

And we remember that we have set $c = 1$. Let's try this equation on for size by setting $u = v = 0.9$ to get

$$w = \frac{1.8}{1.81} \approx 0.99 < 1.0\,. \tag{11}$$

2 Proper Time

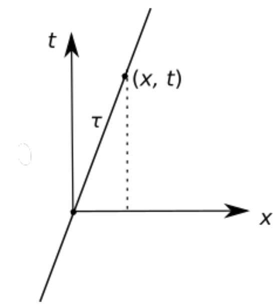

Figure 3. The proper time between two time-like separated events is the time registered by a comoving clock.

The *proper time* τ between event $(0,0)$ and event (x,t) is given by the relation

$$\tau^2 = t^2 - x^2 \,. \tag{12}$$

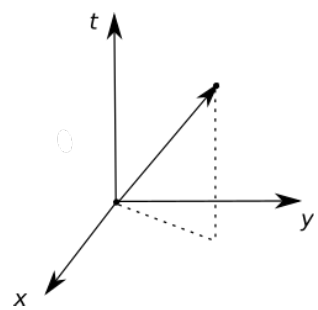

Figure 4. The rotation of coordinates.

By a rotation of coordinates,

$$\tau^2 = t^2 - x^2 - y^2 - z^2 \,. \tag{13}$$

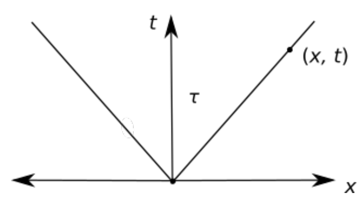

Figure 5. The rotation of coordinates.

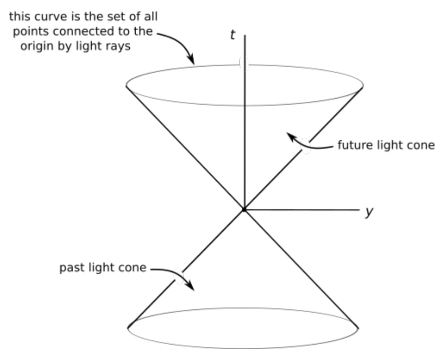

Figure 6. The circle at bottom is the set of all events that can reach the origin at the same time.

Vector in space will be represented in coordinate form to take latin indices as

$$x^i = (x^1, x^2, x^3) \qquad i = 1, 2, 3\,. \tag{14}$$

Vector in spactimee will be represented in coordinate form to take greek indices as

$$x^\mu = (x^1, x^2, x^3) \qquad \mu = 0, 1, 2, 3\,. \tag{15}$$

so,

$$\tau^2 = {x^0}^2 - {x^1}^2 - {x^2}^2 - {x^3}^2\,. \tag{16}$$

3 The derivative by proper time

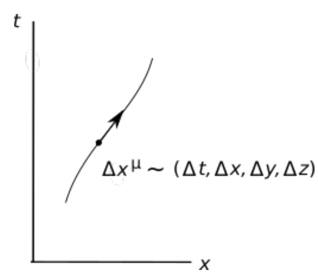

Figure 7. Setting things up to find the proper time derivative.

We ask: What is a 4-D velocity? Let's begin with

$$\frac{\Delta x^\mu}{\Delta \tau} \tag{17}$$

where

$$\Delta\tau^2 = \Delta\tau^2 - \Delta x^2\,. \tag{18}$$

Taken to four dimensions,

$$u^\mu \equiv \lim_{\Delta\tau \to 0} \frac{\Delta x^\mu}{\Delta\tau} \tag{19}$$

where

$$\tau^2 = {x^0}^2 - {x^1}^2 - {x^2}^2 - {x^3}^2\,. \tag{20}$$