String Theory and M-Theory Notes for L. Susskind's Lecture Series (2011), Lecture 1

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July 23, 2023

Abstract

This paper contains my notes on Lecture One of Leonard Susskind's 2011 presentation on String Theory and M-Theory for his Stanford Lecture Series. These read-along notes are meant to aid the viewer in following Susskind's presentation, without having to take copious notes. The fault for all errors in these notes belong solely to me.

1 A Little History

The origin of string theory began long ago (late 1960s) when physicists were studying hadrons (protons, neutrons, mesons)[1], which began even before the quark theory was proposed.

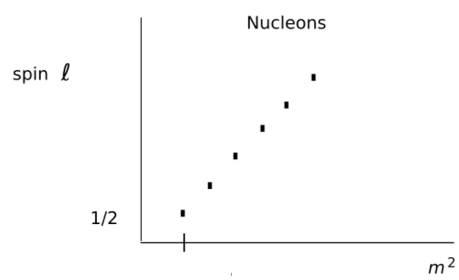

Figure 1a. The Regge trajectories of nucleons.

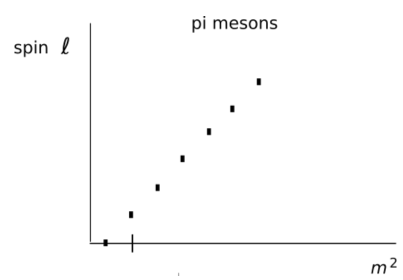

Figure 1b. The Regge trajectories for pi mesons.

[1] A *hadron* is defined as a particle made up of two or more quarks.

This Regge pattern held for many particles, presenting us with the Universal Regge slope for hadrons. The conclusion is that hadrons have structutre, so that they can be up-spun to higher ℓ, which isn't true for electrons. It will be shown that these composite particle can be deformed when spun.

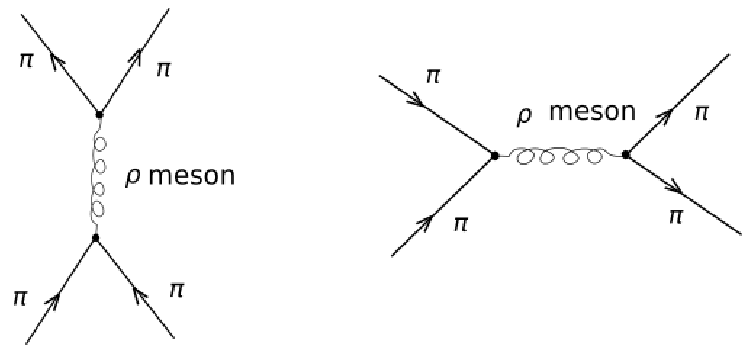

Figure 2. The Regge trajectories for pi meson scattering, with a ρ-meson. as intermediary in a conventional Feymann diagram, as on the left. On the right, we see the diagram for an exchange of a ρ-meson between two pions.

It was found that ρ-meson would appear in various excited states, consistent with the Regge model. And the exchange of the ρ-meson could be added together as well. But when this was taken into account, the two diagrams were not consistent with each other, according to quantum field theory.

Thus, physicists replaced the Feynman diagrams with something less rigorous, but more heuristic.

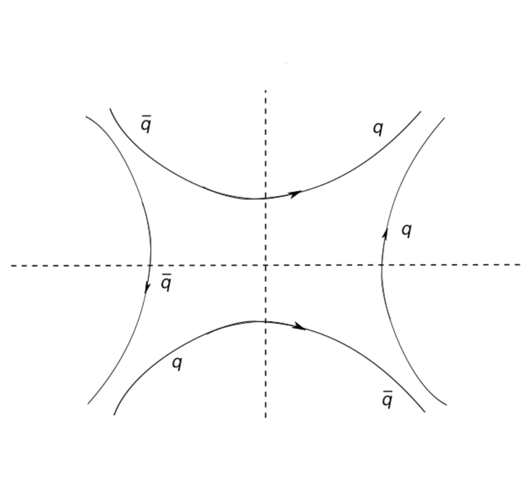

Figure 3. Heuristic diagrams for quark (pi meson) interactions. What holds the quark-antiquark pair together? Maybe some kind of 'string'? By the way, the 'inside' of this structure is referred to as a 'world sheet'.

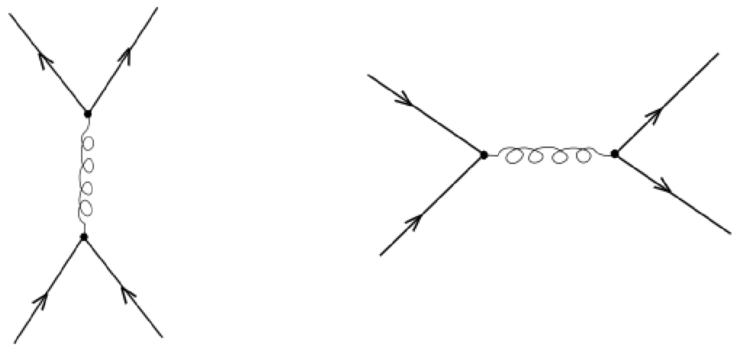

Figure 4. Referring to Fig. 3, by cutting along the horizontal dashed line, we get the figure on the left. And by cutting by the vertical dashed line, we get the Feynman diagram on the right.

So, if there really are quarks, what holds them together to form a meson?

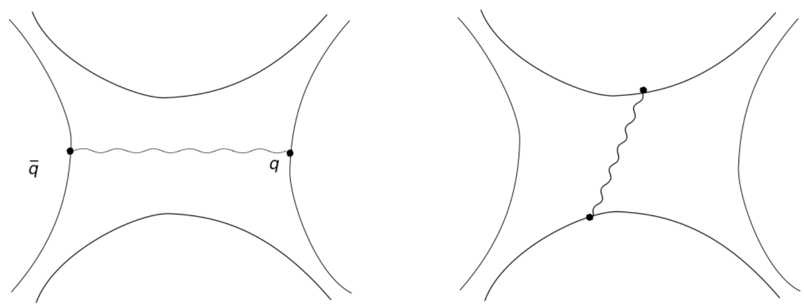

Figure 5. On the left, a hypothetical 'string' connects the quark-antiquark, combination, holding them together. On the right, particle exchange.

And, this is one pathway to modern string theory.

Now, if two quarks really are connected by some string, then the possibility of rotation exists. That is, the pair can rotate about their center of mass. Thus, the string can possess both kinetic energy and potential energy in the form of string stretchiness. So we ask how much the energy will increase as a function of angular momentum?

In the field description of quantum mechanics, we have the gluon field between these two quarks.

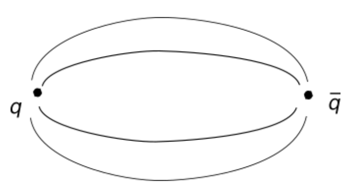

Figure 6. The energy of the quark-antiquark pair is in the field between them. (It looks like an EM field.)

So, the farther the quarks are separated, the weaker the field at a given point. However, the effect in QCD is to bring the field lines together between the two quarks.

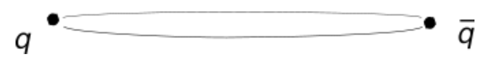

Figure 7. We can think of these strings as being composed of gluons.

2 Kinetics

Though it seems somewhat contradictory to think of constructing a point particle, if we do, we can think of its total enegery as being part kinetic, that is $p^2/2m$ plus a term that represents the energy required to 'assemble' the particle. We'll call this term B (binding energy). Hence,

$$\text{Energy} = \frac{P^2}{2m} + B\,. \tag{1}$$

However, B is an additive constant and independent of P. For a system of many particles, we can write

$$\text{Energy} = \sum_i \Big[\,\frac{P_i^2}{2m_i} + B_i\,\Big]\,. \tag{2}$$

In relativity, we would have the more accurate version of

$$\begin{aligned} E &= \sum_i \sqrt{P^2c^2 + m^2c^4} \\ &= \sum_i \sqrt{m^2c^4\big(1 + \frac{P^2c^2}{m^2c^4}\big)} \\ &= mc^2 \sum_i \sqrt{1 + \frac{P^2c^2}{m^2c^4}} \\ &\approx \sum_i mc^2\big(1 + \tfrac{1}{2}\frac{P^2c^2}{m^2c^4}\big) \\ &= \sum_i mc^2 + \frac{P^2}{2m}\,, \end{aligned} \tag{3}$$

which is the non-relativistic approximation.

3 The Light-Cone Frame

Choose a frame such that along the z-axis the particles have very large momenta. We ask what happens to the energy of such particles? (Set $c = 1$.)

$$E = \sqrt{P^2 + m^2} = \sqrt{P_z^2 + P_x^2 + P_y^2 + m^2}\,. \tag{4}$$

Now, if necessary, boost until all component speeds along the z axis are positive.

$$\begin{aligned} E &= P_z\sqrt{1 + \frac{P_x^2 + P_y^2 + m^2}{P_z^2}} \\ &\approx P_z\Big(1 + \tfrac{1}{2}\frac{P_x^2 + P_y^2 + m^2}{P_z^2}\Big)\,. \end{aligned} \tag{5}$$

If we set $p^2 \equiv P_x^2 + P_y^2$, then

$$E = \sum P_z + \sum \frac{p^2}{2P_z^2} + \frac{m^2}{2P_z^2}\,. \tag{6}$$

But $\sum P_z$ is the constant momentum in the z direction. And, as this term will not be useful for describing the dynamics of the system, it can be dropped!

Now,

$$H = \sum \frac{p_i^2}{2P_{z,i}^2} + \frac{m_i^2}{2P_{z,i}^2}\,. \tag{7}$$

If the RHS terms are small-valued, the system changes slowly in time. This effect can be viewed as SR time dialation. If the energies of a system are small, the changes to the system occur slowly. With respect to the motion in the x, y-plane, the Hamiltonian looks like it's geared for a non-relativistic description of the motion.

Note:

$$H\psi = i\hbar\frac{\partial}{\partial t}\psi\,. \tag{8}$$

We can interpret the term $m_i^2/2P_{zi}$ as a constant and thus we can ignore it. The 'infinite momentum frame' is nowadays referred to as the "Light-Cone Frame." This frame allows us to analyze string problems using non-relativistic quantum mechanics.

Postulate: We will model particles as elastic strings. Furthermore, we will imagine strings to be collections of points. We begin by examining open strings.

So, now we look at the energy equation from a new perspective:

$$\begin{aligned} E &= \sum \frac{m_i\dot{X}_i^2}{2} + \frac{m_i\dot{Y}_i^2}{2} + \text{interaction term} \\ &= m\sum \frac{\dot{X}_i^2}{2} + \frac{k(X_i - X_{i+1})^2}{2}\,, \end{aligned} \tag{9}$$

where X_i is now standing for motions in both the X and Y directions, and we assume all the masses are the same. Therefore, for the Hamiltonian as particle interdistances go small:

$$\mathcal{H} = \int_0^\pi d\sigma\Big[\frac{\dot{X}^2(\sigma)}{2} + \frac{1}{2}\Big(\frac{\partial X}{\partial\sigma}\Big)^2\Big]\,. \tag{10}$$

(The reason for making the upper limit π will become clear in later lectures.) And for the Lagrangian:

$$\mathcal{L} = \int_0^\pi d\sigma\Big[\frac{\dot{X}^2(\sigma)}{2} - \frac{1}{2}\Big(\frac{\partial X}{\partial\sigma}\Big)^2\Big]\,. \tag{11}$$

Note: The parameter k can be chosen so that it can be 'absorbed' away.

Now we consider the special case in which the string center of mass is fixed. If we call this string a particle, then the $\mathcal{H}$ represents all of its internal motion/energy. The position of the particle is assumed to be X_{CM}.

Anyway, by relativistic correspondence,

$$\mathcal{H} = m^2\,, \tag{12}$$

where the m^2 corresponds to the Regge graphs.

We treat the string as a collection of linked springs, whose vibrations are quantized. Then,

$$\left(\frac{\partial X}{\partial \sigma}\right)^2 \sim L^2 \sim m^2 \,. \tag{13}$$

Thus the length of the string, L, is proportional to $\partial X/\partial\sigma$. So, by Hooke's Law, the potential energy goes as the square of this, which is L^2. Nevertheless, by relativity, we know that the energy of the particle is proportional to m, which is proportional to L. So, in the rest frame, Hooke's Law does not apply, and the energy goes as the length.

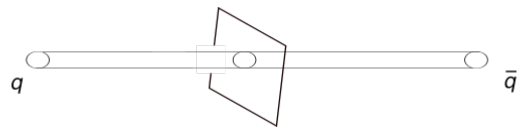

Figure 8. Lines of flux, like electric or magnetic, has uniform strength between the quarks..

The energy density along the tube of flux is uniform (as in a superconductor), so that the total energy is proportional to the length. It's been proven experimentally that the tension internal to hadrons is a fixed value.

The effect is that we have to different models of particle energy depending on which reference frame we look at it from.