String Theory and M-Theory Notes for L. Susskind's Lecture Series (2011), Lecture 10

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Abstract

This paper contains my notes on Lecture Ten of Leonard Susskind's 2011 presentation on String Theory and M-Theory for his Stanford Lecture Series. These read-along notes are meant to aid the viewer in following Susskind's presentation, without having to take copious notes. The fault for all errors in these notes belongs solely to me.

1 Introduction

We study multidimensional toroidal compactification. 3-pairs of sides.

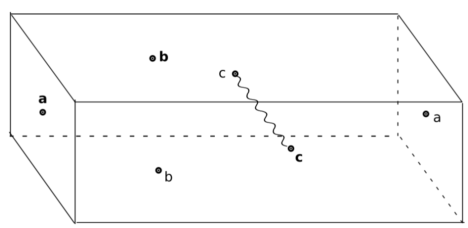

Figure 1. Toroidal compactification with three pairs. Opposite faces identified.

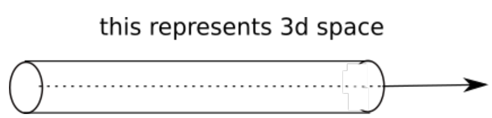

Figure 2. 3-d space with compactified dimension.

For the moment we'll think only of closed, oriented strings.

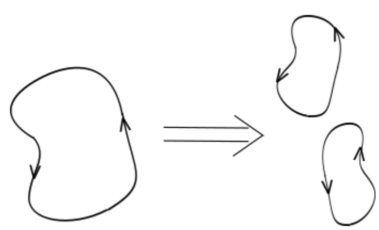

Figure 3. Splitting up maintains orientation.

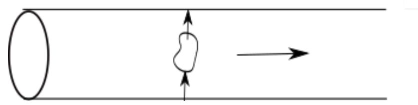

Figure 4. String in motion along the length and around the circumference.

2 Momentum and winding number

The string momentum in the compact dimension is quantized and inversely proprtional to the radius.

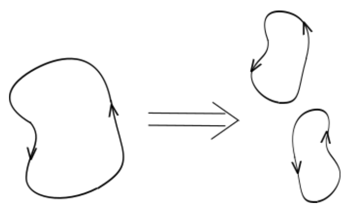

Figure 5. How can strings join?

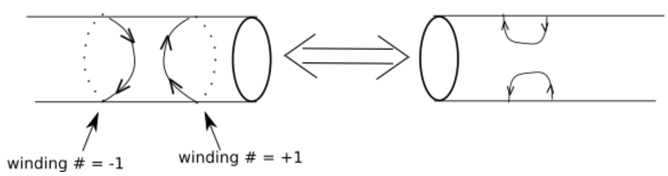

Figure 6. How can strings join or break up?

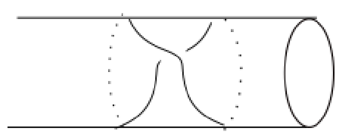

Figure 7. Two strings, each of winding number +1, join to give a winding number of +2. Winding number is a conserved quantity.

Conclusion: The winding number of interacting string is preserved.

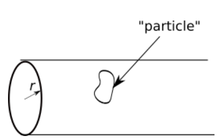

Figure 8. The particle is characterized by it components of momentum.

The component of momentum in the noncompactified direction has to be an integer multiple of a unit quantum of momentum and to be proportional to the radius, n/r. Let x be the coordinate going along the tube. Then

$$\frac{n}{r} = \frac{dx}{d\tau}\,. \tag{1}$$

The energy of the wound string with tension = 1, tension = energy / length

$$E \sim Wr\,, \qquad W = \text{winding } \#\,.$$

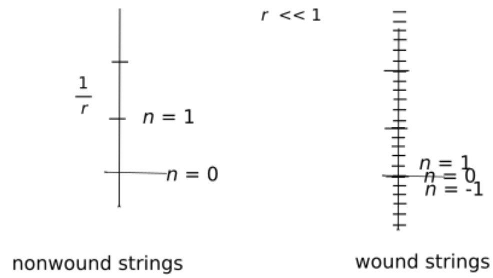

Figure 9. For r very small.

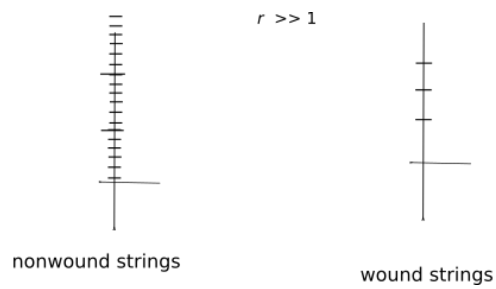

Figure 10. For r very large.

Suppose that the energy of the particle (string) that can be measured is its energy. What is the radius of compactification? Now, it's not possible to disambiguate the energy due to momentum vs the energy due to high winding numnber. We may consider the individual segments of the string as each having momentum so that the total momentum of the string is the sum over each segment.

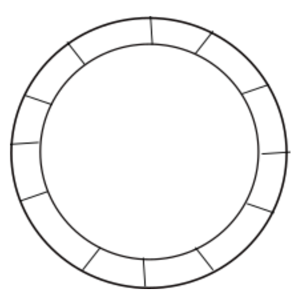

Figure 11. Segmented string moving relative to each other.

$$\sigma \propto \frac{y}{r}\,, \tag{2}$$

Then,

$$W = \frac{1}{r}\int \frac{\partial y}{\partial \sigma} \,. \tag{3}$$

So, T-Duality is equivalent to
1) interchanging $n \leftrightarrow W$,
1) interchanging $r \leftrightarrow 1/r$,
2) interchanging $\dfrac{\partial y}{\partial \tau} \leftrightarrow \dfrac{\partial y}{\partial \sigma}$.

3 Gravitation 5-d spacetime: Kaluza-Klein theory

Now, the winding corresponds to elastic charge. Strings with same same winding orientation repel each other, etc. But where does this attraction/repulsion come from? From gravitation in the 5-dimensional spacetime, with metric g_{MN}

The field A_μ generates both electric and magnetic fields. In 5-D the metric is

$$g_{\mu\nu} \leftrightarrow g_{\mu 5} \oplus g_{55} \,, \tag{4}$$

where $g_{\mu\nu}$ is the usual Einstein metric. Now,

$$g_{\mu 5} \leftrightarrow A_\mu \,, \tag{5}$$

and g_{55} is a scalar field $\Phi \sim$ the radius of the 5th dimension.

$$g_{MN} \sim \begin{array}{c|c} g_{\mu\nu} & A_{\mu 5} \\ \hline A_{\mu 5} & g_{55} \end{array}$$

It's as if there are two kinds of electric fields living side-by-side, corresponding to the winding number orientations.

Winding photons.

Momentum of photons are part of the $g_{\mu 5}$, but what is the field associated?

Spectrum of closed strings: We begin with an unexcited state, and then create excitations of oscillations. 'a' creates a unit of excitation. a's are labeled by direction of space perpendicular to the momentum of the object.

a_n^i if the string is going down the z-axis, where $i = x_1, x_2$ or a compaction in direction y. Also, n is a frequency number of the energy. We also take account of the L/R orientation: $a_n^i(L)$, $a_n^i(R)$

Rule of Level-Matching: The energy in the left-moving wave is equal to the energy in the right-moving wave.

We begin with a hypothetical particle having -2 units of energy (a tachyon). What, then, is the next level of excitation? Is it $a_1^i(L)\,|\,0\,\rangle$ or $a_1^i(R)\,|\,0\,\rangle$? Actually, neither of these conforms to the Rule. The next possibility is to have one L and one R operator. i, j are real space indicies. $a_1^i(L)a_1^j(R)\,|\,0\,\rangle$. But this can produce a graviton. What about $a_1^i(L)a_1^5(R)\,|\,0\,\rangle$? It has a photon-like math structure. What about $a_1^i(R)a_1^5(L)\,|\,0\,\rangle$? On taking the sum of these two possibilities, we get a graviton behavior. But on taking their difference, we get the photon-like field $b_{\mu\nu}$ [the Kalb-Ramond field]. Therefore, $a_1^5(L)a_1^5(R)\,|\,0\,\rangle$ behaves as a scalar and corresponds to Φ.

T-Duality:

$$n \longleftrightarrow \text{winding } \#\,, \tag{6a}$$

$$r \longleftrightarrow \frac{1}{r}\,, \tag{6b}$$

$$\frac{\partial x}{\partial \sigma} \longleftrightarrow \frac{\partial x}{\partial \tau}\,, \tag{6c}$$

$$g_{\mu 5} \longleftrightarrow b_{\mu 5}\,. \tag{6d}$$

However, we don't have candidates for all these possible fields.

4 D-Branes and open strings

D = Dirichlet (though not relevant); 'brane' comes from 'membrane'.

D-Branes are structures necessary for the consistency of string theory. The upshot of this is to establish many equivalences between various string theories (such as Calabi–Yau manifolds).

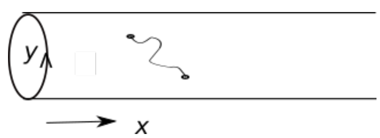

Figure 12. We need to account for Duality on open strings.
Note: open strings don't have winding numbers.

The boundary conditions for an open string are

$$\frac{\partial x}{\partial \sigma} = 0\,, \quad \frac{\partial y}{\partial \sigma} = 0\,, \tag{7}$$

which means that there is no stretching is allowed at the end points.

5 T-Duality equivalence (compact direction only)

In the compact dimension:

$$\text{Momentum} \sim \frac{\partial y}{\partial \tau}. \qquad \text{Winding } \# \sim \frac{\partial y}{\partial \tau} \longleftrightarrow \frac{\partial y}{\partial \sigma}.$$

Neumann boundary conditions $\longrightarrow$ Dirichlet B.C. implies the end points are fixed. Therefore, this theory requires some structures on which the endpoints are forever stuck in the compact directions.

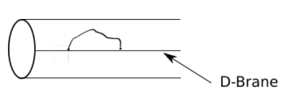

Figure 13. The structure that holds down the endpoints is referred to as a 'D-brane'.

You can decide on the dimension of the brane by choosing the number of constraints. You subtract one dimension of freedom for each constraint imposed.

A D-0 brane is constrained in all directions, which constitutes a new kind of particle.

A D-1 brane is a curve in n-space. If a D-1 brane has its endpoints join, it resembles a string, but it is not the original notion of a closed string. These new strings are much heavier than the orignal strings.

A D-2 brane is called a "membrane." A D3 brane is a 3D space.

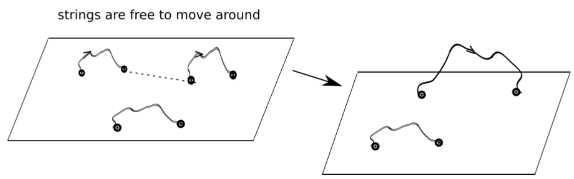

Figure 14. Oriented strings on a brane join at their endpoints.

This begins to look analogous to a Feynman diagram. At low energies, the math of these interactions becomes QFT.

Example: Gluons.

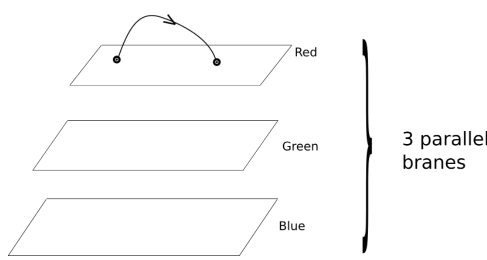

Figure 15. Particles labeled by two colors.

These particles are labeled by pairs of colors: Gluons on strong force.

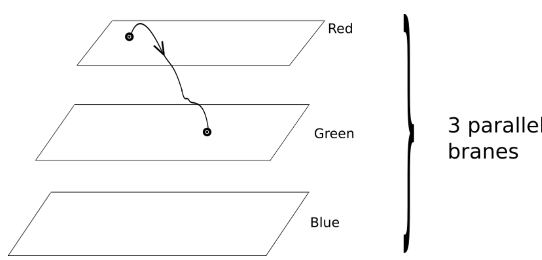

Figure 16. There are eight gluons instead of nine because one linear combination can disappear.

So, what about the quarks? A quark connects to the branes somehow on one end, but the other end goes off to infinity (or to some other).

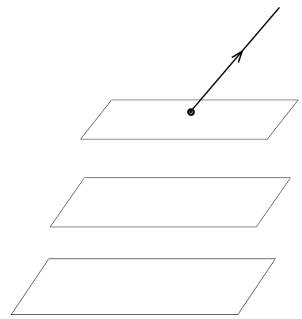

Figure 17. The quarks are exotic.

If two such strings of opposite orientation meet at the connecting points, they can join to form a free-floating string. Annihilation of quark-antiquark.

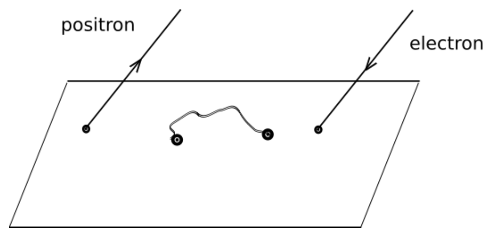

Figure 18. Strings on one brane (similar to QED).

It turns out that a D-string can end on a 3 brane.

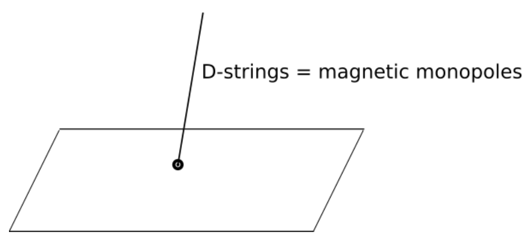

Figure 19. What about the magnetic monopoles? If they exist, they are D-strings and very heavy.

D-branes in 10 dimensions are used to study quantum chromodynamics. And this then takes us back to where the whole thing started from.