String Theory and M-Theory Notes for L. Susskind's Lecture Series (2011), Lecture 5

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October 17, 2023

Abstract

This paper contains my notes on Lecture Five of Leonard Susskind's 2011 presentation on String Theory and M-Theory for his Stanford Lecture Series. These read-along notes are meant to aid the viewer in following Susskind's presentation, without having to take copious notes. The fault for all errors in these notes belong solely to me.

1 Preliminaries

We assume that the nonrelativstic string acts like a spring with energy

$$E = \tfrac{1}{2} k L^2 \,, \tag{1}$$

which is the potential energy. This occurs in our special high-momentum frame in which $m^2 = \frac{1}{2}kL^2$. In the rest frame

$$m = \sqrt{\frac{k}{2}}\, L \approx T L \,. \tag{2}$$

$\sqrt{k}$ is called the *string tension* T.

Next, we set c and $\hbar$ equal to unity, and note the units

$$[\, E \,] = \frac{1}{[\, L \,]} \tag{3}$$

with $[\, k \,] = \text{energy}^2$. But also

$$\omega \approx \sqrt{k} \,. \tag{4}$$

The tension is the restoring force of stretch. The force that the string can endure by stretching is independent of its length.

$$\text{Force is } \frac{\text{energy}}{\text{length}} = \frac{E}{L} = T \,. \tag{5}$$

The shorter the string are, the higher the frequency of oscillation.

Approximate size of a string is one Planck length. How fast does it vibrate? Approximately one Planck time.

2 String Theory Units with Dimensional Analysis

Physics requires 3 units: mass, length, time. We wish to choose units mass according to theoretical needs. There are three universal constraints:

1. The speed of light ($c = 1$)
2. Planck's constant with $\Delta p \Delta x > \hbar$, implies $\hbar = c$.
3. Gravitation applies to everything implies $G = 1$.

Now to combine them

1. $[\, G^p \hbar^q c^r \,] = [\, L^2 \,] = L^2 = \ell^2$.
2. $[\, c \,] = L/T = \ell/t$.
3. $[\, \hbar \,] = [\Delta x\,][\, \Delta p\,] = L[\, m\Delta L/\Delta x\,] = m\ell^2/t$.
4. $[\, G\,][\, r^2/M\,] = \ell^3/mt^2$.

So,

$$\ell^2 = \left(\frac{\ell^3}{mt^2}\right)^p \left(\frac{m\ell^2}{t}\right)^q \left(\frac{\ell}{t}\right)^r = \frac{\ell^{3p+2q+r}\, m^{q/p}}{t^{2p+q+r}}\,. \tag{6}$$

Right away, we know that $p = q$. $p = 1$. After solving for the three parameters, we get

$$\ell \sim \sqrt{\frac{G\hbar}{c^3}} \sim 10^{-35}\,\text{meters}\,. \tag{7}$$

By definition, the Planck time is the time required for light to travel one Planck length,

$$t_p \sim 10^{-43}\,\text{seconds}\,. \tag{8}$$

As for the Planck mass:

$$m_p \sim 10^{-8}\,\text{kg}\,. \tag{9}$$

Note: 1 Planck mass $\sim 10^{19}$ GeV. To get an electron to vibrate at it ground state requires one Planck unit of energy.

3 The Electron

One way to confirm that the electron is a string is to whack it hard and then look for excited states to emerge.

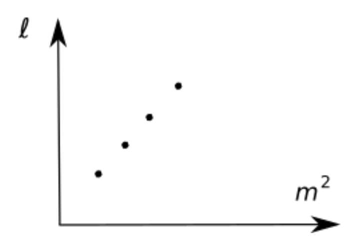

Figure 1. Levels of angular momentum indicate some kind of structure to the electron.

Note: Quantum field theory tells us that the energy scale is about 1000 times the Planck energy.

4 Unification Theory

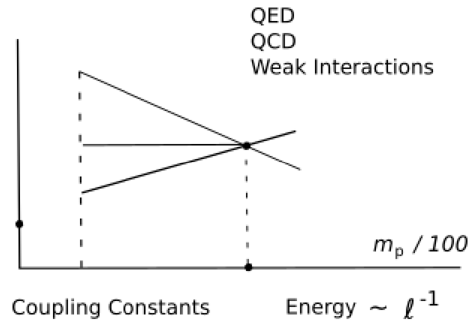

Figure 2. A meeting of the lines. Energies are less than the Planck energy. The string behavior of particles should manifest to the right of the intersection line.

5 Constructing the Massless Photon

To get the photon to turn out right, we start with an open string. The mass squared of the ground state of the string, so that when we employ the creation operator, it comes out massless. So, we set the mass of the ground state to -1.

Harmonic oscillators have zero-point energy. So the ground-state energy of the ground state of a vibrating of a vibrating string..... Zero-point 1/2. The nth oscilllating mode has frequency n, so the ground-state energy of the nth oscillation is $n/2$.

Problem: How does

$$\sum \frac{n}{2} = -1? \tag{10}$$

(The factor of 1/2 comes from $\frac{1}{2}\hbar\omega$.) Let's return to the energy of a very fast moving system. The overall mometum P going down the z axis (in the limit it goes infinitely large). To this we add

$$P + \frac{m^2}{2P} \to E\,, \tag{11}$$

where

$$2P(E-P) = m^2\,, \tag{12}$$

where $2P$ is the time dialation factor.

$$2PE = \underbrace{m^2}_{\text{finite piece}} + \underbrace{2P^2}_{\text{infinite piece}}\,. \tag{13}$$

To use the infinite energy piece to our favor, we need to apply a trick. The 'cheat' here is that, although, the term is infinite, it won't effect the outcome simply because it is constant. So, it turns out that (**??**) should be replaced by the exotic equation

$$\sum \frac{n}{2} = -1 + \infty\,. \tag{14}$$

Let's return to the LHS of (**??**) and just do the sum!

$$\tfrac{1}{2}\sum n = 1 + 2 + 3 + \cdots\,. \tag{15}$$

So, let ϵ be much less than unity. Then

$$Z \equiv e^{-\epsilon} + e^{-2\epsilon} + e^{-3\epsilon} + \cdots \,, \tag{16}$$

yields

$$Z(\epsilon) = \sum_{n=1}^{\infty} e^{-n\epsilon} = e^{-\epsilon} \sum_{n=0}^{\infty} e^{-n\epsilon} = e^{-\epsilon} \sum_{n=0}^{\infty} \left(e^{-\epsilon}\right)^n \,. \tag{17}$$

And for some more trickery:

$$-\frac{\partial Z}{\partial \epsilon} = \sum_{n=1}^{\infty} n e^{-n\epsilon} \,. \tag{18}$$

From this we get that

$$Z(\epsilon) = \frac{e^{-\epsilon}}{1 - e^{-\epsilon}} \,, \tag{19}$$

and

$$\begin{aligned} -\frac{\partial Z}{\partial \epsilon} &= -\frac{\partial}{\partial \epsilon} \frac{e^{-\epsilon}}{1 - e^{-\epsilon}} \\ &= \frac{\partial}{\partial \epsilon} \frac{1}{\epsilon} \frac{1 - \epsilon + \epsilon^2/2 + \cdots}{1 - \epsilon/2 + \epsilon^2/6 - \cdots} \,. \end{aligned} \tag{20}$$

Thus,

$$\begin{aligned} -\frac{\partial Z}{\partial \epsilon} &\approx -\frac{\partial}{\partial \epsilon} \frac{1}{\epsilon} \big(1 + \epsilon/2 + \epsilon^2/2\big)\big(1 + \epsilon/2 - \epsilon^2/6 + \epsilon^2/4\big) \\ &\approx -\frac{\partial}{\partial \epsilon} \frac{1}{\epsilon} \big(1 - \epsilon/2 + \epsilon^2/12\big) \\ &\approx \frac{1}{\epsilon^2} - \frac{1}{12} \,, \end{aligned} \tag{21}$$

where we will take ϵ to zero.

The infinite term $1/\epsilon^2$ gets absorbed into the P^2 term. With the $1/2$ factor we get $-1/24$. We get this value for both the a and b oscillation (x and y oscillations). Hence, the value is back to $-1/12$. To get to -1, we need more degrees of freedom for oscillations. To end up with 24 degrees total: 22 from 11 dimensions and 2 from the (z, t) dimension, making 26.

For the closed loop, we have a -2 state to raise. But we have two ways around each loop (in addition to the a and b), totaling $48/24 = 2$. To proceed further, we need conformal invariance.