Statistical Mechanics Notes for L. Susskind's Lecture Series, Part 1

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Abstract

This is my notes on Chapter One of Leonard Susskind 2013 presentation on Statistical Mechanics for his Stanford Lecture Series. (He also made a 2009 video series on the same topic.) They can be found on Youtube. This lecture sets forth the probability theory we'll need for statistical mechanics.

1 Getting Started

Statistical mechanics is founded on probability theory. In the discrete case, in which we can index by the variable i, we have the following rules to be obeyed:

$$P(i) \geq 0\,, \tag{1a}$$

$$\sum_i P(i) = 1\,, \tag{1b}$$

$$\lim_{N\to\infty} \frac{N_i}{N} = P(i)\,. \tag{1c}$$

We interpret N_i/N as the ratio of the number of items that can be in the ith state compared with the number of items that can be in any state.

We define the *average* of $F(i)$ to be

$$\langle\, F\,\rangle = \sum_{i=1}^{n} F(i)P(i)\,. \tag{2}$$

In my own notes, if a quantity is an average, I'll usually indicate it by applying an overbar

$$\bar{F} = \langle\, F\,\rangle\,. \tag{3}$$

Entropy

In thermodynamics, we have two kinds of variables: intrinsic and extrinsic. The former do not scale as the volume or mass scales, but the latter do. An example of which is the density, which is the ratio of mass to volume. Extrinsic variables do scale as the mass or volume. For example, if you double the mass of an object, you double its weight. Most common variables in thermodynamics are extrinsic (unless they are converted to per unit mass or volume, etc). For our purposes, we want entropy S to be an extrinsic variable, so that, if the number of possible states goes up, the entropy goes up, too.

Let A be a system divided into two independent subsystems A_1 and A_2. Let Ω_1 be the number of states of subsystem A_1 and let Ω_2 be the number of states of subsystem A_2. By the rules of probability, the number of state of A is their product $\Omega_1\Omega_2$. We look for a function H such that if we multiply states, the entropy is the sum of the individual numbers of states.

$$H(\Omega_1) + H(\Omega_2) = H(\Omega_1\Omega_2)\,. \tag{4}$$

Fortunately, we know a simple function that has this property – the logarithm.

$$\log(\Omega_1) + \log(\Omega_2) = \log(\Omega_1\Omega_2)\,. \tag{5}$$

So what is the entropy? Mathematically speaking, it's a statistical measure of the number of states of a system.
Liouville's Theorem tells us that the number of enclosed states in phase space is a constant.

Zeroth Law of Thermodynamics

The zeroth law of thermodynamics states that if two thermodynamic systems are each in thermal equilibrium with a third system, then they are in thermal equilibrium with each other. (Wikipedia)

First Law of Thermodynamics

The total energy of an isolated system is conserved.

Therefore, if systems A and B are allowed to share energy with each other over time, and if the change in energies of A is dE_A and B is dE_B over a small time frame, then

$$\frac{dE_A}{dt} = -\frac{dE_B}{dt}\,. \tag{6}$$

The most convenient definition of entropy for our use is

$$S = -\sum_i P(i)\log P(i)\,, \tag{7}$$

for the discrete case, and

$$S = -\int P(p,x)\log P(p,x)dp\,dx\,, \tag{8}$$

for the continuous case, integrated over phase space.